MATH PROBLEM OF THE WEEK

Fall 2022

Problem 7

For a positive integer $\mathbf{n}$, let $S_{\mathbf{n}}$ be the subset of $\{1, 2, \ldots, \mathbf{n}\}$ which consists those numbers $\mathbf{k}$ for which 1 is the only common divisor of $\mathbf{n}$ and $\mathbf{k}$. Let $\alpha(\mathbf{n})$ denote the average of the numbers in $S_{\mathbf{n}}$. For example when $\mathbf{n} = 14$ then $S_{14} = \{1, 3, 5, 9, 11, 13\}$ and

$$\alpha(14) = \frac{1+3+5+9+11+13}{6} = 7.$$

Find a formula for $\alpha(\mathbf{n})$ in terms of $\mathbf{n}$ and justify your answer.

We welcome solutions from everyone. The undergraduate participant from the NMSU main campus with the most correct solutions at the end of the semester will receive an award of $500.

Solutions must be mathematically rigorous and originally obtained by the participants. Participants will be notified if their solutions are correct within a week.

Deadline: Monday, November 14, 10 am

Next problem will be posted on November 14

Send solutions to: mathpotw@nmsu.edu

More information at: https://math.nmsu.edu/activities/math-problem-of-the-week.html