NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 6

Fall 2022

Problem. If M and N are positive integers such that $M + MN + N = 1146$ then what is the value of

$$M + N?$$

HINT: $1147 = 31 \times 37$.

Solution. By adding 1 to the given equation, we get

$$
\begin{aligned}
M + MN + N + 1 &= 1146 + 1 \\
(M + 1)(N + 1) &= 1147.
\end{aligned}
$$

Since 1147 is the product of two primes, namely 31 and 37, either

$$M + 1 = 31 \ \& \ N + 1 = 37$$

$$\text{or } M + 1 = 37 \ \& \ N + 1 = 31.$$

Thus, either $(M, N) = (30, 36)$ or $(M, N) = (36, 30)$, and in both cases

$$M + N = 66.$$

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