NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 1

Fall 2023

Problem 2

If a and b are two real numbers such that $a+b=\sqrt{14}$ and $a-b=\sqrt{12}$. Then find the value of

$$x = 2ab(a^2+b^2).$$

Solution. If $a+b=\sqrt{14}$ means

$$(a+b)^2 = a^2+b^2+2ab = 14,$$

and $a-b=\sqrt{12}$ implies

$$(a-b)^2 = a^2+b^2-2ab = 12.$$

If we set $\mathsf{X} = a^2+b^2$ and $\mathsf{Y} = 2ab$, then the above equations imply

$$\begin{aligned} \mathsf{X}+\mathsf{Y} &= 14 \\ \mathsf{X}-\mathsf{Y} &= 12. \end{aligned}$$

By solving these equations we get $\mathsf{X} = 13$ and $\mathsf{Y} = 1$, and therefore

$$x = 2ab(a^2+b^2) = \mathsf{XY} = 13.$$

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