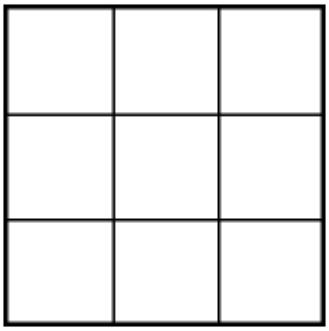
NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 2

Fall 2022

Problem. Dexter wants to plot 10 points on a square paper of dimension $3\mathsf{cm} \times 3\mathsf{cm}$ such that distance between any two points is strictly greater than $\sqrt{2}\mathsf{cm} \approx 1.41421\mathsf{cm}$. If possible, how? If not, why?

Solution. Such an arrangement is impossible. To see this, notice that $3\mathsf{cm} \times 3\mathsf{cm}$ paper consists of nine $1\mathsf{cm} \times 1\mathsf{cm}$ square regions. Thus, if we plot ten points, there exists two points which are in the same box.



In a $1\mathsf{cm} \times 1\mathsf{cm}$ square region any two points are at a distance less than or equal to the length of the diagonal which is $\sqrt{2}$. Therefore, the 2 points which are in the same $1\mathsf{cm} \times 1\mathsf{cm}$ square region are at a distance less than or equal to $\sqrt{2}$.