## NMSU MATH PROBLEM OF THE WEEK

## Solution to Problem 2

Spring 2022

## Problem 2

If the intersection of the rectangle ABCD and EFGH is a square, then find the perimeter of the rectangle AFGD.


Solution. Although we do not know the width of the rectangle ABCD and EFGH, the diagram suggests they are equal, call it $\alpha$. Then each side of the square EBCH equals $\alpha$. Therefore,

$$
\begin{aligned}
& |\mathrm{BF}|=|\mathrm{EF}|-|\mathrm{EB}|=|\mathrm{HG}|-|\mathrm{HC}|=30-\alpha \\
& |\mathrm{DH}|=|\mathrm{DC}|-|\mathrm{HC}|=|\mathrm{AB}|-|\mathrm{EB}|=45-\alpha
\end{aligned}
$$

and

$$
\begin{aligned}
\text { perimeter of } \mathrm{AFGD} & =|\mathrm{AB}|+|\mathrm{BF}|+|\mathrm{FG}|+|\mathrm{GH}|+|\mathrm{HD}|+|\mathrm{DA}| \\
& =45+(30-\boldsymbol{\alpha})+\boldsymbol{\alpha}+30+(45-\boldsymbol{\alpha})+\boldsymbol{\alpha} \\
& =150 .
\end{aligned}
$$

Note that the value of $\alpha$ cannot be determined. However, we can calculate the perimeter of AFGD because it is independent of $\alpha$.

