NMSU MATH PROBLEM OF THE WEEK

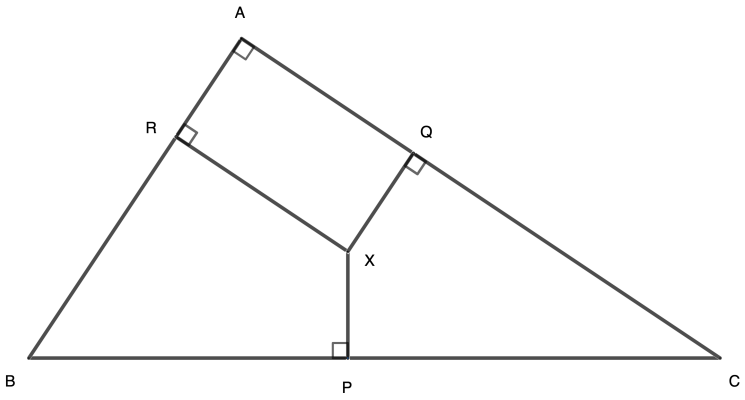
Solution to Problem 4

Fall 2021

Problem 4.

Let $\triangle ABC$ be a right triangle with $\angle A = 90^\circ$ and X an arbitrary point inside the triangle. Let P, Q, and R be the feet of the perpendicular lines from X to BC, AC, and AB, respectively. Prove that

$$BP^2 + CQ^2 + AR^2 = CP^2 + BR^2 + AQ^2.$$



Solution.

By Pythagorean theorem we have:

$$\begin{aligned} XR^2 + AR^2 &= XA^2 = XQ^2 + AQ^2 \\ XP^2 + BP^2 &= XB^2 = XR^2 + BR^2 \\ XQ^2 + CQ^2 &= XC^2 = XP^2 + CP^2. \end{aligned}$$

The solution now follows by adding these equations and canceling $XR^2 + XP^2 + XQ^2$ at both sides of the equation.