

NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 6

Fall 2021

Problem 6.

For a positive integer n , let t_n be the number of positive divisors of n . Prove that

$$t_1 + t_2 + \cdots + t_n = \left\lfloor \frac{n}{1} \right\rfloor + \left\lfloor \frac{n}{2} \right\rfloor + \cdots + \left\lfloor \frac{n}{n} \right\rfloor.$$

Note: Here $\lfloor x \rfloor$ denotes the floor function, that is, the largest integer N such that $N \leq x$.

Solution.

For each integer $1 \leq a \leq n$, the number of integers in the range $[1, n]$ that are divisible by a is $\left\lfloor \frac{n}{a} \right\rfloor$. On the other hand, a is counted on the left hand side of the equation precisely on those t_i for i a multiple of a . The conclusion follows.