

A stochastic model for ontogenetic growth

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Abstract: Based on the mechanism of biomass inhibition underlying energy conversion throughout an organism's lifespan, numerous mathematical models have been developed to study ontogenetic growth patterns of animals. While deterministic equations have provided valuable insights into growth dynamics, real-world animal growth often deviates from strict determinism due to stochastic factors like genetic variation and environmental fluctuations. In this study, we extend Geoffrey et al.'s (2001) deterministic model by incorporating stochasticity using white noise.

Drawing from a comprehensive dataset compiled by Geoffrey et al., comprising 206 data samples from 13 species of animals exhibiting both determinate and indeterminate growth patterns, we employ a two-step approach. Initially, we fit the average growth trajectory to a deterministic curve. Subsequently, we incorporate stochasticity by identifying the best-fitted white noise term to represent growth variance. By combining these two fits, we obtain a stochastic differential equation (SDE) model that improves the prediction of growth trajectories, capturing the inherent variability of biological systems. The resulting SDE model generates a band for each species, with a central curve representing average biomass and a width indicative of mass variance over time. Furthermore, we propose a universal curve capable of encapsulating growth patterns across diverse species. This framework also holds promise for extending insights to other domains such as plant growth and tumor development, where genetic and environmental factors play pivotal roles in shaping trajectories.