More Efficient Computation of the Economic Equilibrium

Gustav Rubio[1], Elise Wilcher[1], Nataliya Kalashnykova[2],
Olga Kosheleva[1], and Vladik Kreinovich[1]
[1]University of Texas at El Paso, El Paso, TX 79968, USA
grubio5@miners.utep.edu, emwilcher@miners.utep.edu,
olgak@utep.edu, vladik@utep.edu
[2]Universidad Autónoma de Nuevo León (UANL)
San Nicolás de los Garza, Nuevo León, C.P. 66455 Mexico
nkalash2009@gmail.com

Formulation of the practical problem. After each change in the economic situation, market economy eventually settles on some equilibrium state. However, often, there are wild oscillations preceding this settling, when the producers either over- or underproduce – which, in both cases, negatively affects their bottom line. To avoid such oscillations, it is desirable to set up production levels $q_1, \ldots, q_n$ corresponding to the equilibrium. So, it is practically important to compute these equilibrium values.

What is known. There are known formulas for computing the equilibrium; see, e.g., [1]. Let $D(p)$ denoted the dependence of the demand D on the price p. Each producer is supposed to have a hypothesis describing how the price p will change if this producer changes its production level, i.e., how the derivative $v_i \stackrel{\text{def}}{=} -\dfrac{\partial p}{\partial q_i}$ depends on q_i. Also, for each producer, we know how the production cost depends on the production level q_i; in practice, it is sufficient to use a quadratic approximation to this dependence: $b_i \cdot q_i + (1/2) \cdot a_i \cdot q_i^2$. Under these assumptions, in the equilibrium, the following equations are satisfied:

$$\sum_{i=1}^{n} q_i = D(p); \quad v_k(q_k) = \frac{1}{\sum\limits_{i \neq k} \dfrac{1}{v_i(q_i) + a_i} - D'(p)} \text{ for all } k.$$

We thus have $n+1$ equations to determine the $n+1$ unknowns $p, q_1, \ldots, q_n$.

Computational challenge and what we do in this talk. For large n, solving a system of $n+1$ nonlinear equations is computationally complicated. In this talk, we show that from the computational viewpoint, we can reduce this problem to solving a single equation with one unknown. Solving a single equation is much easier, so this reduction can save a lot of computation time.

Our reduction. If we divide 1 by both sides of second equilibrium equation and add $1/(v_k + a_k)$ to both sides, we conclude that $\dfrac{1}{v_k} + \dfrac{1}{v_k + a_k} = C$, where $C \stackrel{\text{def}}{=} \sum\limits_i \dfrac{1}{v_i + a_i} - D'(p)$. If we multiply both sides of this equality by $v_k \cdot (v_k + a_k)$, we get a quadratic equation, based on which we can get an explicit expression for $v_k(q_k)$ in terms of C. Now that we know how all the values v_i depend on C, we can use the definition of C to describe how $D'(p)$ depends on C. Then, by applying a function which is inverse to $D'(p)$, we can get an expression for p in terms of C.

Similarly, by applying the inverse function v_i^{-1} to $v_i(q_i)$, we can get an expression for each q_i in terms of C. Substituting the expressions for q_i and p in terms of C into the first equilibrium equality, we get the desired equation with one unknown C. Once we solve this equation and find C, we can use the explicit expressions for p and q_i in terms of C to compute the desired equilibrium.

[1] J. G. Flores Muñiz, N. Kalashnykova, V. V. Kalashnikov, and V. Kreinovich, *Public Interest and Public Enterprize: New Developments*, Springer, Cham, Switzerland, 2021.