A Pre-Bohr Explanation of the Periodic Table: How Could a Wrong Theory Fit Data So Well?

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Formulation of the problem. Usually, a good fit between a theory and the data means that the theory is true. However, there was a known exception. In the early 20 century, John Nicholson showed that the atomic weight w of each element from the periodic table can be represented – with accuracy 0.1 – as an integer combination of 4 basic weights: $w_1 = 0.51282$, $w_2 = 1.008$, $w_3 = 1.6281$, and $w_4 = 2.3615$: $w = n_1 \cdot w_1 + n_2 \cdot w_2 + n_3 \cdot w_3 + n_4 \cdot w_4$ for some integers $n_i \geq 0$. This led him to a conclusion that all atoms consists of combinations of 4 basic particles with these weights. The fit was perfect – but the theory turned to be wrong. How can it be?

What is known and what we do. In [1], we have shown that for the specific 4 weights w_i selected by Nicholson, *any* number larger than $n_0 = 3.03$ – and not just the atomic weights – can be represented, with accuracy 0.1, as an integer combination of these 4 weights. In this talk, we show that a similar property – maybe for some other n_0 – holds for any selection of small random weights w_i. Specifically, we show it on a toy example when all 4 selected weights w_i are close to 1. Similar arguments – with a different n_0 – work when we have some values w_i close to 2.

Our explanation. To get the overall weight close to an integer n, we need to add n values which are close to 1. So, we must have $n_1 + n_2 + n_3 + n_4 = n$, where n_i is the number of times we pick w_i. Each tuple (n_1, n_2, n_3, n_4) can be graphically described if we place n 1s in a row, and add dividers after the first n_1 1s, after $n_1 + n_2$ 1s, and after $n_1 + n_2 + n_3$ 1s. This way, possible tuples are in 1-1 correspondence with selecting 3 numbers out of $n + 3$. So, there are $N = \binom{n+3}{3} = \frac{(n+3) \cdot (n+2) \cdot (n+1)}{1 \cdot 2 \cdot 3}$ such tuples.

Since we picked 4 random weights w_i, it is reasonable to conclude that we have N numbers randomly distributed around n – and since we have no reason to assume that some values are more probable or that different sums are dependent, it makes sense to assume that all values are equally probable and independent, i.e., that we have N independent uniformly distributed random variables. For each value $x \approx k$, the only possibility of not being approximated with accuracy 0.1 by one of the N sums is when all N sums lie outside the interval $[x - 0.1, x + 0.1]$ of width 0.2. The probability for each sum to be outside this interval is equal to $1 - 0.2 = 0.8$, so the probability for all N sums to be outside is 0.8^N. To get this probability smaller than 0.05, we need $0.8^N \leq 0.05$, i.e., $N \geq 13.4$, which happens for $n \geq 4$, so in this case $n_0 = 4$. If we want the probability smaller than 0.1%, we need $N \geq 30.9$, which happens starting with $n = n_0 = 5$.

[1] O. Kosheleva and V. Kreinovich, "Why was Nicholson's theory so successful: an explanation of a mysterious episode in 20 century atomic physics", *Mathematical Structures and Modeling*, 2021, Vol. 60, pp. 39–43.