Polynomial Dedekind Domains

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A groundbreaking result of Claborn proves that any abelian group is the class group of a Dedekind domain. He shows how to construct such a Dedekind domain in a polynomial ring with infinitely many variables. A few years later, Eakin and Heinzer gave a more tractable approach to show that any finitely generated abelian group G is the class group of a Dedekind domain between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$; they show that the intersection of $\mathbb{Q}[X]$ with a suitable finite family of DVRs of $\mathbb{Q}(X)$ which are residually algebraic extension of $\mathbb{Z}_{(p_1)}, \dots, \mathbb{Z}_{(p_n)}$, where $p_1, \dots, p_n \in \mathbb{Z}$ are primes, is Dedekind with class group G.
We show here that a ring in Eakin and Heinzer's construction can be realized as a ring of integer-valued polynomials, and, more generally, we give a full characterization of the Dedekind domains between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$, called Polynomial Dedekind domains. Furthermore, we prove that any group which is the direct sum of a countable family of finitely generated abelian groups occurs as the class group of a Polynomial Dedekind domain. This result is sharp, in the sense that every Polynomial Dedekind domain has class group of this form.
The result is obtained by a concrete description of residually algebraic torsion extensions of $\mathbb{Z}_{(p)}$ to $\mathbb{Q}(X)$ by means of a suitable kind of pseudo-convergent sequences in a fixed algebraic closure of the field of p-adic numbers $\overline{\mathbb{Q}_p}$.
In particular, we also obtain a full characterization of the PIDs between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$.

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