Capstone Mathematics from Primary Historical Sources

David Pengelley

New Mexico State University

Course Overview

The senior-level course *Great Theorems: The Art of Mathematics* implements the philosophy of learning mathematics directly from primary historical sources [1,2,3,7,9]. The central focus is always great mathematics, but since the approach is entirely through interconnected historical sources, students learn a great deal of history at the same time. Each offering of the course studies one or more sequences of (translated) primary sources at the upper undergraduate level, each sequence following a great theme in mathematics over centuries or millennia, leading to one or more great results in modern mathematics. The aim is for students to experience a sequence of discovery close to firsthand.

Adopting the view of mathematics as art, we examine the creation of some mathematical masterpieces from antiquity to the modern era. A careful analysis of these theorems and their original proofs illustrates the aesthetic spirit which pervades mathematics; a comparison with modern theorems and methods through demanding exercises demonstrates the progress mathematics achieves through abstraction. And a mathematical term paper on an approved topic chosen through the student's initiative rounds out a capstone experience.

We also place the theorems in a larger historical context, since mathematics is an inseparable part of human history, effecting profound changes in people's lives, and itself responding to new social and political needs. Finally, as these theorems were discovered by real flesh-and-blood people, it is only appropriate to look at how their lives and work were influenced by the intellectual and social environment of the day.

For most students, the course is a huge breath of fresh air and an eye-opening capstone experience. For mathematics majors, the exposure to deep mathematics through its historical development is a great way to round out and enrich their major and make connections between their prior courses. For the mathematically inclined engineering and science majors who qualify for this course, it is a refreshing and stimulating alternative to the often heavily cookbook-oriented mathematics courses required by their major, and it leaves them with a much happier and more positive intellectual view of mathematics.

Great Theorems: The Art of Mathematics began in 1988 as an honors mathematics course attracting students majoring in mathematics, engineering, computer science, or the physical sciences at New Mexico State University, and has been offered every year since then [5]. The prerequisite is a year of calculus and some upper division mathematics course, all with good grades. Students are expected to contribute actively to class discussion in a course with enrollment restricted to twenty.

The course qualifies as a senior elective for either the major or minor in mathematics, the major in secondary mathematics education, and as an upper-division general education breadth elective for engineering majors. Usually about half the students are mathematics majors, with the rest from secondary mathematics education, engineering, computer science or the physical sciences.

Course Design

Our primary resource is a collection of historical texts, ideally with generous annotation, commentary, and exercises. The goal is to study the original proofs of the theorems in these texts, in the words of the discoverers, in order to understand the most authentic possible picture of the evolution of major branches of mathematics during a span of over two thousand years. It is exciting and illuminating to read original works in which great mathematical ideas were first revealed, and we aim for lively discussion involving everyone.

At home and in class we read, discuss, and interpret the theorems and their proofs, with students writing their thoughts and questions about these works, and we discuss how the various sources tie together in the development of major mathematical results. Regular written assignments based on the primary sources consist of proving related results, filling in missing parts of proofs, and related historical questions. One of our main aims is to have students actually do mathematics themselves by creating some ideas and devising proofs on their own.

Classroom pedagogy varies by instructor. I have a three-part non-lecture assignment approach for each day of my teaching, explained in a handout for students (Appendix C), and in more detail in [8]. Two of these parts are pre-class, and one is post-class. For each day, I expect students first to read new material well before class, and to write questions about their mathematical reading to give to me to read before class. Second, I expect students to prepare pre-assigned mathematical problems to bring to class based on their reading. These two pre-class assignments are graded based only on preparation, with a quick plus, check, or minus.

In class we begin by discussing their reading questions. Class discussions are often challenging, because primary sources provide fabulous grist for deep and wide-ranging considerations. Today they frequently raise as many questions as they answer, a fabulous pedagogical tool. Then most of class time is spent with students working together on the previously assigned problems, along with whole-class discussions or student presentations suggested by me when interesting questions and approaches arise.

Finally, the third part of each day's assignment occurs post-class, consisting of final homework on the topic, a very few challenging exercises not tackled in class. Students are encouraged to discuss their ideas with others, but are then expected to finish and write up their polished post-class homework entirely on their own, in their own words, to hand in for me to read and mark carefully. I often request rewriting for improvement. This post-class homework part ultimately receives a single letter grade for quality, one for each class day.

The course grade is based on a final holistic evaluation of student work: roughly one half on daily assignments (i.e., student writings on the primary sources, and related mathematical assignments); roughly one quarter on class participation; and the remaining quarter on a term paper and a brief oral presentation on it, as described below. There are no exams. The course overview handout for students is in Appendix B.

The course has a very flexible timetable (hence there is no course outline appended), often influenced by what explorations happen in the classroom based on student response and activities, according to the classroom methods described above. Whether we engage one topic chapter or two from the textbook during a term varies.

On the first day I introduce and discuss the nature and expectations of the course, we dive into some mathematics, and I ask students to skim the entire four topic chapter materials of the textbook as homework, to provide feedback, as explained earlier, at the beginning of the second class period. At the beginning of the second day I select a topic on the spot, based on their feedback, and we begin right away with primary source material from that topic. Succeeding days always have reading/writing in advance, preparatory mathematical work on exercises, in class group and whole class work, and final homework exercises, as described earlier. By mid-term students begin work on individual term paper research, in addition to our regular classroom work, as described earlier. The very end of term is spent on short term paper presentations and discussion.

Resources

When I first co-created and taught the course in 1988, we had little more than a handful of chosen primary sources on a few topics, some of which we had to translate ourselves, often with no annotation, context, or exercises. We assigned some essay readings to supplement the sources, created assignments as we went, and started writing annotation to tie it all together. And indeed this is how anyone can still develop their own materials; I enthusiastically recommend it. Guidance on the pedagogical principles, and on design of materials, can be found in [1,2,3,7,9]. Today primary sources are much more easily available, and in translation as well, than when we started. The reader may be pleasantly surprised that finding promising and appropriate primary sources for teaching on a given topic is not as hard as may be feared. The bibliography [10] provides a window to many historical sources for teaching. The recent sizeable source book [12] would be a good place to find many good sources on a variety of topics (see the review [11]).

As the course evolved, with new instructors developing their own sequences of primary sources on new topics, we coalesced on sequences of primary sources for four great themes in the evolution of mathematics leading to important modern results, added extensive annotation, contextual, historical, and mathematical commentary as a guide and overview of each big story, and numerous mathematical exercises for students. We also included copious references to the literature for deeper understanding by both teachers and students. These four sequence themes became the four chapters of the course textbook *Mathematical Masterpieces: Further Chronicles by the Explorers* [4]. Each chapter has an extensive introduction, which tells the story of a large theme from the beginning, both mathematically and historically. As it proceeds, the chapter introduction points the reader to the subsequent chapter sections, which focus on sources by specific authors. Therefore we have students read the introduction in tandem with work on individual chapter sections, going back and forth between the main story and the featured primary source sections.

Each one-semester offering of the course typically covers only one or two of the four independent chapter themes described below. The chapters are available individually online from the publisher.

The chapter themes and the authors of the primary sources they include are:

- *The Bridge Between Continuous and Discrete.* Primary sources follow two millennia of the theme of sums of numerical powers. Archimedes sums squares to find the area inside a spiral, Fermat and Pascal sum powers using figurate numbers and binomial expansions, Jakob Bernoulli discovers the pattern in sums of powers formulas, and Euler develops his summation formula for making astonishing approximations by summing divergent series, and solves the Basel problem that the sum of the reciprocal squares is $\pi^2/6$.
- *Solving Equations Numerically: Finding Our Roots.* Qin solves a fourth degree equation, Newton develops a proportional method, Simpson explains a fluxional method ("Newton's Method"), and in 1981 Smale proves that probabilistically the fluxional method almost always converges.
- *Curvature and the Notion of Space.* In primary sources Huygens discovers the isochrone, Newton derives the radius of curvature, Euler studies the curvature of surfaces, Gauss defines an independent notion of curvature, and Riemann explores higher dimensional space.
- *Patterns in Prime Numbers: The Quadratic Reciprocity Law.* Euler discovers patterns in prime divisors of quadratic forms, Lagrange develops a theory of quadratic forms and divisors, Legendre asserts the quadratic reciprocity law, Gauss proves it, Eisenstein creates a geometric proof, and Gauss composes quadratic forms, foreshadowing the class group.

As illustrations, Appendix A introduces small excerpts from selected primary source material for each theme, along with connected sample exercises for students. The website [6] provides sample sections from each chapter.

Assignments

Regular homework and related classroom work are the heart of the course. Assignments are largely mathematical in nature, based directly on the primary sources, since the course is first and foremost mathematics, set authentically in its history. Exercises often strengthen students' understanding of a primary source, and are sometimes open-ended. To give a diversity of flavors, Appendix A provides sample exercises from our four general themes, each exercise preceded by a little context and a small excerpt from the relevant primary source.

Regarding the term paper and brief oral presentation (see the handout for students in Appendix D), the choice of topic is up to the student, subject to my approval, but should include a meaningful mathematical component (it should not be mostly biographical) that they can genuinely understand and explain to others in a presentation. I do not suggest topics, so students must keep their eyes open for something along the way of interest, since this is an opportunity to delve into something personally exciting or innovative. I consciously place this responsibility on each student, to try to encourage them to take initiative. This results in a great variety of quality in topics, and while not all are inspiring, many are truly fascinating

Students are expected to pick a term paper topic by mid-semester, and I help students in refining ideas for a topic. I first ask each student to come up with two ideas for a topic, to do a preliminary library search to see that adequate research materials are to be found there (required usually to be books, not just internet sources), and to write a paragraph describing each topic to me, along with references to what was found in the library. After possible further refinement, I approve each topic. I then sometimes require that students show me their writing progress along the way, to help them complete an acceptable paper on time. We use the mandatory final exam period, as well as the last few class days as needed, for term paper presentations, with papers due several days before presentations.

Lessons Learned

This is an engaging capstone course for seniors who have studied a good amount of upper-level mathematics. It greatly broadens their horizons, ties together other course material, and simply excites most of them no end. The material is thrilling, and most of them love learning from primary sources.

The class discussions and work are often exhilarating for students and instructor alike; this course is probably my favorite, most beloved, repeating course of all time to teach.

However, it is important, as with any senior-level mathematics course, that students have sufficient strength entering the course to succeed. Since the only specific content prerequisite is a year of calculus, insisting on the requirement of some upper-division mathematics with good grades is essential to assure that students are all at the necessary level and can succeed.

Even with good mathematical preparation, students will vary in their ability to tackle primary sources and contribute to class discussions and presentations. The instructor may need to spend extra time assisting and encouraging some, draw some out in discussion and presentation, and pair them up in class with other students who can be helpful in a teaching capacity.

A final caveat: It is highly rewarding to find and incorporate one's own primary sources, but they vary tremendously in their pedagogical value and appropriateness. Some are gems, whereas some I have aimed for have turned out to be too opaque for any kind of classroom use. And the amount of each source to use, and how, requires much thought. Some of these issues are addressed further in [3].

References

1. J. Barnett, J. Lodder, and D. Pengelley, The pedagogy of primary historical sources in mathematics: classroom practice meets theoretical frameworks, *Science & Education,* 23 (2014) 7-27.

2. J. Barnett, J. Lodder, and D. Pengelley, Teaching and learning mathematics from primary historical sources, *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 26 (2016) 1-18.

3. J. Barnett, J. Lodder, D. Pengelley, I. Pivkina, and D. Ranjan, Designing student projects for teaching and learning discrete mathematics and computer science via primary historical sources, in *Recent Developments in Introducing a Historical Dimension in Mathematics Education,* V. Katz and C. Tzanakis, eds., Mathematical Association of America, Washington, D.C, 2011, 189-201.

4. A. Knoebel, R. Laubenbacher, J. Lodder, and D. Pengelley, *Mathematical Masterpieces: Further Chronicles by the Explorers,* Springer Verlag, New York, 2007. Excerpts and reviews at http://www.math.nmsu.edu/~history/.

5. R. Laubenbacher and D. Pengelley, Mathematical masterpieces: teaching with original sources, in *Vita Mathematica: Historical Research and Integration with Teaching,* R. Calinger, ed., Mathematical Association of America, 1996, 257-260. Also at http://www.math.nmsu.edu/~history/.

6. R. Laubenbacher and D. Pengelley, *Teaching with Original Historical Sources in Mathematics,* a resource web site, http://www.math.nmsu.edu/~history/.

7. R. Laubenbacher, D. Pengelley, and M. Siddoway, Recovering motivation in mathematics: Teaching with original sources, *Undergraduate Mathematics Education Trends,* 6, No. 4 (September, 1994), parts of pages 1,7,13 (and at http://www.math.nmsu.edu/~history/).

8. David Pengelley, *Comments on Classroom Dynamics*, 2010, at http://www.math.nmsu.edu/~davidp/.

9. David Pengelley, Teaching with primary historical sources: Should it go mainstream? Can it?, in *Recent Developments in Introducing a Historical Dimension in Mathematics Education,* V. Katz and C. Tzanakis, eds., Mathematical Association of America, Washington, D.C., 2011, 1-8. Also at http://www.math.nmsu.edu/~davidp/.

10. David Pengelley, Some selected resources for using history in teaching mathematics, 2011, http://www.math.nmsu.edu/~history/resources.html. This list includes numerous collections of primary sources, collected works, historically oriented mathematics books, articles on teaching using history, historically oriented teaching materials, reference works, specialized and general histories, periodicals, and web resources.

11. David Pengelley, book review of *Mathematics Emerging: A Sourcebook 1540-1900*, by Jacqueline Stedall, *Notices, American Mathematical Society,* 58 (2011) 815-819.

12. Jacqueline Stedall, *Mathematics Emerging: A Sourcebook 1540-1900*, Oxford University Press, Oxford, 2008.

Appendix A: Sample primary source materials and exercises

Here I describe a small primary source excerpt and related exercise for each major theme we have used in teaching the course.

- *The Bridge Between Continuous and Discrete*

As part of never-ending discoveries of the interplay between the continuous and the discrete, Euler found the Euler-Maclaurin summation formula, which among other things enables incredibly accurate approximations for the sums of very slowly converging series.

"The general expression that we found in the previous chapter for the summative term of a series, whose general term corresponding to the index x is z, namely

$Sz=\int zdx+(1/2)z+(\mathfrak{A}dz)/(1\cdot 2dx)-(\mathfrak{B}d^3z)/(1\cdot 2\cdot 3\cdot 4dx^3)+(\mathfrak{C}d^5z)/(1\cdot 2\cdots 6dx^5)$-etc.,

actually serves to determine the sums of series whose general terms are integral rational functions of the index x, because in these cases one eventually arrives at vanishing differentials. On the other hand, if z is not such a function of x, then the differentials continue without end, and there results an infinite series that expresses the sum of the given series up to and including the term whose index =x. The sum of the series, continuing without end, is thus given by taking x=∞, and one finds in this way another infinite series equal to the original."

Exercise: Follow in Euler's footsteps by using his summation formula to approximate $\sum_{i=1}^{\infty} 1/i^3$ to seventeen decimal places. Euler obtained 1.20205690315959428. Study whether it appears to be a simple rational multiple of π^3, as Euler hoped it might be.

- *Solving Equations Numerically: Finding our Roots*

What we now call "Newton's Method" for approximating roots is actually due to Simpson, who developed the method, with examples, for not just one, but two unknowns.

"When there are two Equations given, and as many Quantities (x and y) to be determined. Take the Fluxions of both the Equations, considering x and y as variable, and in the former collect all the Terms, affected with $\dot{x}$, under their proper Signs, and having divided by $\dot{x}$, put the Quotient = A; and let the remaining Terms, divided by $\dot{y}$, be represented by B: In like manner, having divided the Terms in the latter, affected with $\dot{x}$, by $\dot{x}$, let the Quotient be put = a, and the rest, divided by $\dot{y}$, = b. Assume the Values of x and y pretty near the Truth, and substitute in both the Equations, marking the Error in each, and let these Errors, whether positive or negative, be signified by R and r respectively: Substitute likewise in the Values of A, B, a, b, and let $\frac{Br-bR}{Ab-aB}$ and $\frac{aR-Ar}{Ab-aB}$ be converted into Numbers, and respectively added to the former Values of x and y; and thereby new Values of those Quantities will be obtained; from whence, by repeating the Operations, the true Values may be approximated ad libitum."

Exercise: Formulate Simpson's fluxional method geometrically for three functions each in three variables. Tangent planes now become tangent "hyerplanes." Derive his formulas or their modern counterparts.

- *Curvature and the Notion of Space*

Gauss, building on work of Euler on principal curvatures, developed a general theory of curvature of surfaces.

"The solution of the problem, to find the measure of curvature at any point of a curved surface, appears in different forms according to the manner in which the nature of the curved surface is given. When the points in space, in general, are distinguished by three rectangular coordinates, the simplest method is to express one coordinate as a function of the other two. In this way we obtain the simplest expression for the measure of curvature. But, at the same time, there arises a remarkable relation between this measure of curvature and the curvature of the curves formed by the intersections of the curved surface with planes normal to it. Euler, as is well known, first showed that two of these cutting planes which intersect each other at right angles have this property, that in one is found the greatest and in the other the smallest radius of curvature; or, more correctly, that in them the two extreme curvatures are found. It will follow then from the above mentioned expression for the measure of curvature that this will be equal to a fraction whose numerator is unity and whose denominator is the product of the extreme radii of curvature. The expression for the measure of curvature will be less simple, if the nature of the curved surface is determined by an equation in x, y, z. And it will become still more complex, if the nature of the curved surface is given so that x, y, z are expressed in the form of functions of two new variables p, q."

Exercise: Look up Gauss's formula for the curvature of a surface given in terms of two parameters, p, q [87, p. 18], and use this to compute the curvature of the torus in Exercise 3.19 at an arbitrary point.

- *Patterns in Prime Numbers: The Quadratic Reciprocity Law*

After many decades of struggle, in 1772 Euler gave a clear statement of his final vision, still not proven, of the role reversal [reciprocity] between quadratic residues [residues] and moduli [divisors].

"Conclusion. These … theorems, of which the demonstration from now on is desired, can be nicely formulated as follows:

Let s be some prime number, let only the odd squares 1, 9, 25, 49, etc. be divided by the divisor 4s, and let the residues be noted, which will all be of the form 4q+1, of which any may be denoted by the letter α, and the other numbers of the form 4q+1, which do not appear among the residues, be denoted by some letter $\mathfrak{A}$, then we shall have

divisor a prime number [P] of the form	*then [modulo P]*

4ns+α	*+s is a residue, and -s is a residue;*
4ns-α	*+s is a residue, and -s is a nonresidue;*
4ns+$\mathfrak{A}$	*+s is a nonresidue, and -s is a nonresidue;*
4ns- $\mathfrak{A}$	*+s is a nonresidue, and -s is a residue."*

Exercise: Use Euler's claims in his Conclusion of 1772 about the quadratic character of –s to find the linear forms of nontrivial prime divisors of numbers of the quadratic form $x^2 + 5y^2$.

Appendix B: Handout for students on course overview

GREAT THEOREMS:

THE ART OF MATHEMATICS

Required Text: *Mathematical Masterpieces: Further Chronicles by the Explorers,* available at the University Bookstore.

Prerequisites: Grades of B or better in second semester calculus, and any upper division MATH/STAT course, with overall GPA of 3.2 or better, or consent of instructor. Please speak with me if you do not have the prerequisites.

Course grade:

Written homework assignments:	50%
Term Paper:	25%
Class Participation:	25%

The ancient Greeks counted arithmetic and geometry among the Seven Liberal Arts. Adopting the view of mathematics as art, we will examine the creation of some mathematical masterpieces from antiquity to the modern era. A careful analysis of these theorems and their original proofs will illustrate the aesthetic spirit which pervades mathematics; a comparison with modern theorems and methods will demonstrate the progress mathematics achieves through abstraction.

We will also place the theorems in a larger historical context, since mathematics is an inseparable part of human history, effecting profound changes in people's lives, and itself responding to new social and political needs. Finally, as these theorems were discovered by real flesh-and-blood people, it is only appropriate to look at how their lives and work were influenced by the intellectual and social environment of the day.

The text contain the primary sources and theorems we will study. We have endeavored to provide the original proofs of these theorems, in order to present the most authentic possible picture of the evolution of mathematics during a long span of time. In class we will discuss and interpret these theorems and their proofs, and we hope for lively discussion involving everyone. As we examine the development of mathematical ideas, we will also discuss their historical context and the biographies of their creators. Regular written assignments based on the original mathematical sources will consist of proving related results, filling in missing parts of proofs, and related historical questions. One of our main aims is to have you actually do mathematics yourself by creating some ideas and devising proofs on your own.

In addition to the regular assignments there will be a term paper based on library research into a mathematical topic you will choose with instructor advice and approval. As the semester goes along, be looking for a topic for your paper, perhaps related to what we do in class. By midsemester you should have chosen the paper’s topic. Near the end of the semester, after your written paper is finished, each person will give a brief class presentation to tell everyone else what they explored and discovered.

Appendix C: Handout for students on homework assignment guidelines

Keep this sheet

Guidelines for all regular homework assignments

Please put your name (and any nickname your prefer) on the first page, *staple* your pages together, and *do not* fold them. Use both sides of the paper if you wish, to save paper. Please *do not* write in light pencil. Please write clearly. Thank you.
Parts A, B, C of each homework are equally important.
Part A: Advance preparation. Hand this in at the beginning of class, one class period before our class discussion and work on new reading. Reading responses (a), questions (b), reflection (c), and time spent (d):
You do *not* need a new page for each part (a),(b),(c),(d).

a) Read assigned material. Reread as needed for complete understanding. Then write clear *responses* to assigned questions about the reading.
b) Write down some of your own explicit *questions* about your reading, ready to bring up in class. This may involve new or old concepts that are confusing to you, and connections to other ideas. You should also consider writing down what was well explained and interesting, what was confusing, what you had to reread but eventually understood.
c) Reflection: Write two or three sentences *reflecting* on the process of your work; this should only take a few minutes. Write about how things went with any assignment or reading done for class, and other course work. This should reflect both your ongoing personal feelings about the course as a whole and your interaction with the material at hand.
d) Write how much *time* you worked on part A.

Part B: Warmup exercise preparation to present in class. This is due during class when we begin to discuss new material. Work individually, and then with others in your group outside class time, on a few assigned easy warmup exercises on the new material we will discuss, based on your advance reading in Part A. Write up the solutions to these individually, to hand in in class. I will ask individuals and groups to present some of these to the class, to get us started discussing new material. Be sure to hand these in before leaving class.
Also always write how much time you worked on part B, and with whom.

Part C: Main exercises. These will be assigned after class discussion and work on new material. They will normally be due next period. Work individually and with others in your group on these. Also come to see me during office hours or at other appointment times about these. I am happy to help you. Then go home and write up your final solutions completely by yourself, without comparing with other people. The paper you hand in should be entirely your own writing, not the same as anyone else's.

Appendix D: Handout for students on term paper guidelines

Guidelines for term paper and presentation

One quarter of your work for the course is a well written term paper on a mathematical topic of your choice, along with a brief class presentation.

- You should take the initiative in finding a term paper topic. First come up with at least two ideas for topics, do a preliminary library check that adequate research materials are likely to be found there, and write a paragraph describing each topic, along with what you found in the library. Borrow the library books you might need now, before someone else does. Use your imagination and interests in selecting topics! Hand this in by the date assigned in class.
- The principal requirement for a topic is that it should be about mathematics. The other main requirement is that you should be able to discuss the mathematics in your paper with some genuine understanding of it. Writing a paper that lists mathematical results you have no understanding of is not fruitful. The paper should be written in a style understandable by your fellow students.
- After you hand in your topic paragraphs, you will receive feedback on your tentative topics, and may be asked to seek further source material in the library to make sure a topic is appropriate. Your selected topic should then receive final approval.
  - Your paper should be well written in your own words. It should include a complete list of detailed references, and frequent citations to your references, indicating page numbers from your references. I expect you will use at least 2 or 3 references, and that at least some will come from the library. If you wish to quote directly from a source somewhere in your paper, instead of writing something in your own words, you should indicate clearly that it is a quotation. References from the internet should also be cited with full information on the exact internet location of the information, and a copy of an internet printout attached. If you copy directly from a source without attribution this is plagiarism, and you will fail the course. Any consistent format for the paper and references is acceptable.
  - Your paper should be 6-12 pages long.
  - You may handwrite or type your paper clearly. Don't waste time trying to type mathematics or drawings. If you type it, please use 1½ or double spacing.
  - You are strongly urged to consult about your work in progress, and to submit a rough draft before the final version of your paper. These steps will greatly enhance the likely quality of your paper.
  - When you hand in your paper, by the date assigned in class, keep a copy to prepare for your presentation to the class. Please **do not** put your paper in a plastic cover. Just staple it! Thanks.