

NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 4

Spring 2022

Problem 4.

An equilateral triangle Γ can be covered with five equilateral triangles of side x . Prove that Γ can be covered with four of the triangles of side x .

Solution.

Let ℓ be the side length of Γ . Consider the following six points: the three vertices and the three midpoints of the sides of Γ , and notice that any two of these points are at least $\frac{\ell}{2}$ units apart. Since five of the triangles of side x cover Γ , by the Pigeonhole Principle we must have that two of the six points belong to one of the triangles. Therefore, $x \geq \frac{\ell}{2}$.

To finish the proof we note that the six points considered above divide Γ in four triangles of side $\frac{\ell}{2}$. Thus, since $x \geq \frac{\ell}{2}$, a triangle of side x can cover any one of these triangles.