- **Hardy-Littlewood maximal function.** Given $f \in L_1(\mathbb{R})$ define

$$M(f)(t) = \sup_{t \in I} \frac{1}{|I|} \int_I |f(s)| ds, \quad t \in \mathbb{R},$$

  where I denotes an interval $\mathbb{R}$ and $|I|$ the length of I.

- **Maximal ergodic function.** Let T be a contraction on $L_p(\Omega)$ for every $1 \leq p \leq \infty$. Form the ergodic averages of T

$$A_n = \frac{1}{n+1} \sum_{k=0}^{n} T^k$$

  and define

$$M(f)(t) = \sup_{n \geq 0} \left| A_n(f) \right|.$$

These maximal functions satisfy the following inequality: For $1 < p \leq \infty$

$$\left\| M(f) \right\|_p \leq C_p \|f\|_p, \quad \forall f \in L_p(\mathbb{R}) \text{ or } L_p(\Omega),$$

where C_p is a constant depending only on p. This classical result is due to Hardy-Littlewood and Dunford-Schwartz, respectively.