NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 7

Fall 2023

Problem 7

Prove that for some integer $k > 1$, 3^k ends with 0001 (in its decimal representation).

Solution: Consider the infinite collection $\{3^k \mod 10000 : k \geq 1\}$ of all possible last 4 digits of 3^k. Since there are only finitely many possibilities, we must have some $m > n > 1$ such that 3^m and 3^n have the same last 4 digits. In other words,

$$3^m = 3^n \mod 10000.$$

Because 3^n and 10000 are coprime, we can cancel the 3^n to reach

$$3^{m-n} = 1 \mod 10000.$$

The number $k = m - n > 1$ satisfies 3^k ends with 0001 in its decimal representation.