NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 3

Fall 2024

Problem 3

Show that at any party there are two people who have the same number of friends at the party. (Assume that all friendships are mutual.)

Solution. Let n be the number of people at the party, and suppose that all n attendees have a distinct number of attending friends. Since the least number of friends possible is 0 and the most number of friends possible is $n-1$, there are n possible numbers of friends. Hence we must have an attendee with each number k of friends for all values $0 \leq k \leq n-1$. In particular, there is someone with $n-1$ friends (friends with everyone) and someone with 0 friends (friends with none). This is a contradiction, so we conclude that there are two people with the same number of friends in attendance.