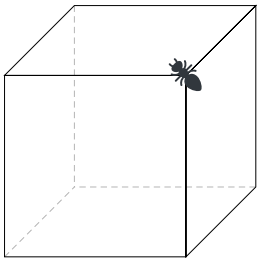
NMSU MATH PROBLEM OF THE WEEK

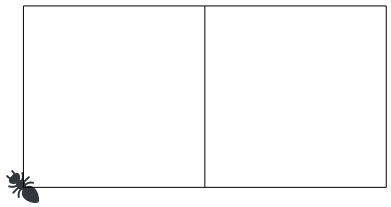
Solution to Problem 2

Spring 2025

An ant begins at one corner of a cube and walks along the edges of the cube without backtracking or returning to any corner it has previously visited. In how many ways can it travel to the opposite corner?



Solution. At the beginning the ant has three choices, which are permuted by a symmetry of the cube. We will suppose that the ant travels straight down and multiply our final answer by 3. Next the ant has two choices, which are exchanged by a symmetry of the cube which does not move the first edge the ant walked down. We will assume the ant travels left and multiply our answer by 2. The five edges touching the two vertices the ant has already been at are now off-limits, and after flattening the remaining portion of the cube we have the following situation, where the ant's destination is the bottom center point.



By direct observation, we now see that there are 3 ways for the ant to traverse from the bottom left to the bottom center: right, up right down, or up right right down left. In total we have $3 \cdot 2 \cdot 3 = 18$ possibilities.