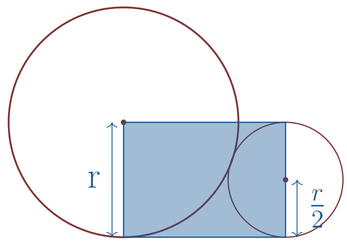
NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 3

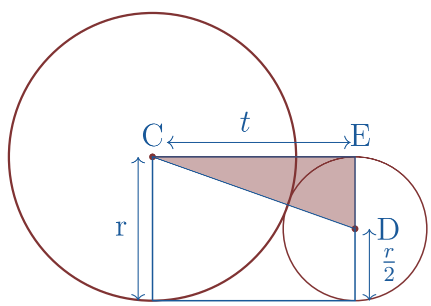
Spring 2025

In the following diagram, the circle of radius r and a circle of radius $\frac{r}{2}$ meet each other tangentially. Find the area of the shaded rectangle as a function of radius r.



Justify your answer.

Solution. We already know that the length of the vertical side of the rectangle is r. To find the length of the horizontal side, call it t, we join the center of the given circles and consider the right-angled triangle ΔCDE as shown in the following diagram.



Since the line CD passes through the point of contact of the two circles, we have

$$|\mathrm{CD}| = r + \frac{r}{2} = \frac{3r}{2},$$

and the length of DE equal to $\frac{r}{2}$. Therefore, by Pythagoras theorem

$$t = |\mathrm{CE}| = \sqrt{|\mathrm{CD}|^2 - |\mathrm{DE}|^2} = \sqrt{\left(\frac{3r}{2}\right)^2 - \left(\frac{r}{2}\right)^2} = \frac{\sqrt{8}}{2}r = \sqrt{2}r.$$

Thus, the area of the shaded rectangle equals $rt = \sqrt{2}r^2$.