

# NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 1

Fall 2022

**Problem.** If  $f$  is a function from  $\mathbb{R}$  to  $\mathbb{R}$  such that

$$f(x)f(y) = f(x+y) + f(x-y)$$

and  $f(1) = 3$ , then what is the value of  $f(7)$ ?

**Solution.** By setting  $x = 1$  and  $y = 0$ , we get

$$f(1)f(0) = f(1) + f(1),$$

thus  $f(0) = 2$ . Now by setting

- $x = 1, y = 1$ , we get  $f(1)f(1) = f(2) + f(0)$ , thus  $f(2) = 7$ ,
- $x = 2, y = 1$ , we get  $f(2)f(1) = f(3) + f(1)$ , thus  $f(3) = 18$ ,
- $x = 3, y = 1$ , we get  $f(3)f(1) = f(4) + f(2)$ , thus  $f(4) = 47$ ,
- $x = 4, y = 3$ , we get  $f(4)f(3) = f(7) + f(1)$ , thus  $f(7) = 843$ .