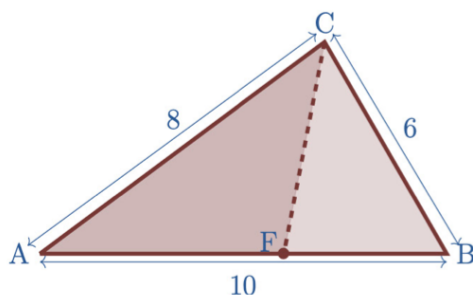# NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 1

Fall 2023

Problem 1

Suppose $\triangle ABC$ is a triangle with sides of length $|AB| = 10$, $|BC| = 6$, and $|AC| = 8$. Find the length of the segment AF so that the area of $\triangle AFC$ is twice that of triangle $\triangle FBC$. Justify your answer.

Solution. Area of a triangle is given by $\frac{1}{2} \times \mathsf{base} \times \mathsf{height}$. Note that $\triangle AFC$ and $\triangle FBC$ have the same height. Therefore, in order for $\triangle AFC$ to have double the area of $\triangle FBC$, the length of the base of $\triangle AFC$, which is $|AF|$, must double the base of $\triangle FBC$, which is $|FB|$. In other words,

$$|AF| = 2|FB|.$$

But we also know $|AF| + |FB| = 10$. Thus

$$
\begin{aligned}
|AF| + |FB| &= 10 \\
2|FB| + |FB| &= 10 \\
3|FB| &= 10 \\
|FB| &= \frac{10}{3}.
\end{aligned}
$$

Therefore the length of AF equals $2|FB| = \frac{20}{3}$. ∎

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