# NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 5

**Spring 2025**

Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a function such that $f(bx + a) = x$ and $f(x) = ax + b$. Then find the values of $a$ and $b$. Justify your answer.

**Solution.** Since $f(x) = ax + b$, therefore

$$x = f(bx + a) = a(bx + a) + b = abx + a^2 + b = x.$$

By comparing coefficients, we get

$$ab = 1 \tag{1}$$

$$a^2 + b = 0. \tag{2}$$

From (2), we get $b = -a^2$, and using this in (1), we conclude

$$\begin{aligned} &\Rightarrow \quad a(-a^2) = 1 \\ &\Rightarrow \quad a^3 = -1 \\ &\Rightarrow \quad a = -1. \end{aligned}$$

Thus $b = -a^2 = -1$.