

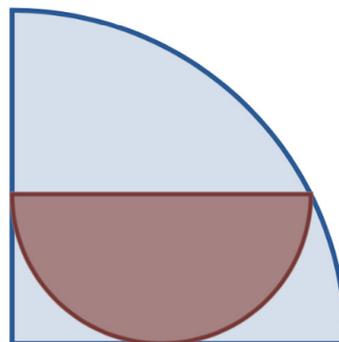
NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 6

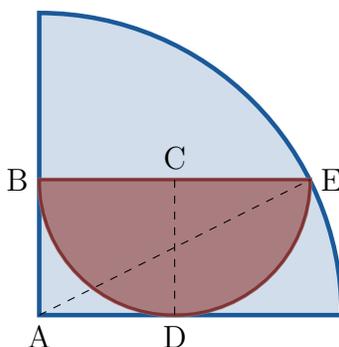
Fall 2023

Problem 6

If the area of the semi disk is 25, then what is the area of its circumscribing quarter disk? Justify your answer.



Solution. First, we make a few constructions as shown in the diagram below:



Observe that ABCD is a square, and therefore in $\triangle ABE$,

- (i) $\angle ABE$ is 90° ,
- (ii) $AB = r$ (radius of the semi disk),
- (iii) $AE = R$ (radius of the quarter disk), and
- (iv) $BE = 2r$.

Thus, by using Pythagoras theorem we get $AB^2 + BC^2 = r^2 + (2r)^2 = R^2 = AE^2$, or equivalently $R^2 = 5r^2$. We are given that $\frac{1}{2}\pi r^2 = 25$, and therefore,

$$\text{Area of the quarter disk} = \frac{1}{4}\pi R^2 = \frac{5}{4}\pi r^2 = \frac{10}{4}\left(\frac{1}{2}\pi r^2\right) = \frac{10}{4}(25) = \frac{125}{2} = 62.5.$$

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