

NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 7

Spring 2023

Problem 7

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Prove that in each year, the 13th day of some month occurs on a Friday.

Solution. First assume that the year is not a leap year. Then the 13th day of the months form the 13th, 44th, 72nd, 103rd, 133rd, 164th, 194th, 225th, 256th, 286th, 317th and 347th day of the year. These numbers modulo 7 (as there are seven days in a week) are

6, 2, 2, 5, 0, 3, 5, 1, 4, 6, 2, and 4

which covers all possible remainders modulo 7. This means regardless of the year there is a month in which the the 13th day is a Friday.



For example, if December 31 of the last year is a Tuesday, and given that the current year is not a leap year, then June 13 of the current year is a Friday. This is because Fridays are n days from a Tuesday if and only if $n \equiv 3 \pmod{7}$.

For a leap year, 13th day of the months are 13th, 44th, 73nd, 104rd, 134rd, 165th, 195th, 226th, 257th, 287th, 318th and 348th day of the year. These numbers modulo 7

6, 2, 3, 6, 1, 4, 6, 2, 5, 0, 3, and 5

also covers all possible remainders modulo 7. Thus, the same is true for leap years as well. ■