

NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 1

Spring 2024

Problem 1

Simplify

$$x = \frac{1}{\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}$$

as a rational number. Justify your answer.

Solution: We first consider the number $y = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$. Notice that

$$y^2 = 6 + \sqrt{6 + \sqrt{6 + \dots}} = 6 + y.$$

Therefore, we have $y^2 = 6 + y$.

Solving this quadratic equation gives two solutions: $y = 3$ or $y = -2$. But notice that y being a square root implies that $y > 0$. Therefore $y = 3$ and thus $x = \frac{1}{y} = \frac{1}{3}$.