



# MATH PROBLEM OF THE WEEK

## Spring 2024

### Problem 9

A set  $S \subset \mathbb{Z}^2$  is called balanced if for each point  $(x, y) \in S$ , exactly two of its 4 adjacent points  $(x \pm 1, y)$  and  $(x, y \pm 1)$  belong to  $S$ . For example,  $S = \{(0, 0), (0, 1), (1, 1), (1, 0)\}$  is balanced while  $S = \{(0, 0), (0, 1), (0, 2)\}$  is not. Find all positive integers  $n$  such that there exists a balanced set of  $n$  elements.

**We welcome solutions from everyone. The undergraduate participant from the NMSU main campus with the most correct solutions at the end of the semester will receive an award of \$500.**

Solutions must be mathematically rigorous and originally obtained by the participants. Participants will be notified if their solutions are correct within a week.

**Deadline: Monday, April 29, 10 am**

This is the last problem of Spring 2024 and carries points equivalent to three problems. We will return in September 2024.

**Send solutions to: [mathpotw@nmsu.edu](mailto:mathpotw@nmsu.edu)**

**More information at: <https://math.nmsu.edu/activities/math-problem-of-the-week.html>**