NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 9

Spring 2025

Suppose 2025 people play in a singles tennis tournament. Is the number of people who have played an odd number of games even irrespective of the structure of the tournament? If so, give a proof. If not, provide a counterexample.

Solution. Yes, the number of players who played odd number of games is even. Before we prove the statement, note that there is no restriction on the structure of the tournament. The only information we have is this is a singles tournament, which means exactly two person play one game.

Let g_n be the number of games played by the n-th player. Then

$$g_1 + g_2 + \cdots + g_{2025} \quad = \quad 2 \times \{\text{number of games played}\}$$

is an even number. If odd number of players play odd number of games then $g_1 + g_2 + \cdots + g_{2025}$ will be an odd number which is a contradiction.