# NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 7

Spring 2025

A football field is completely covered with red, blue, and green paint. Must it be the case that there are a pair of points 1 meter apart which are painted the same color? Justify your answer.

Solution. We will show that there must be such a pair of points. Consider all equilateral triangles of side length 1m. If any two vertices of such a triangle share a color we are done, so suppose that every such triangle has vertices of all 3 different colors. Now consider two such triangles which share a side. Since both triangles use all 3 colors, the two points which are not on the shared side are the same color. For example, if the shared vertices are blue and red, then the other two vertices are green:
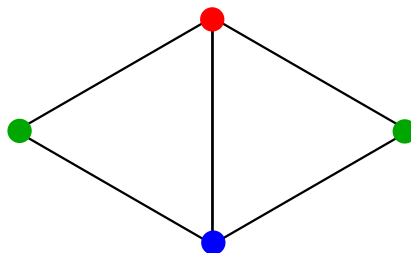
Note that the two green points in this figure have a distance of $\sqrt{3}$ between them. In particular, if we take any two points at a distance of $\sqrt{3}$, then we can see they are the same color by considering the interposing pair of triangles as above. Now consider a circle of radius $\sqrt{3}$ (noting that a football field is large enough to fit such a circle). All points on this circle are the same color as the center point. Taking a chord of length 1m gives two endpoints which are the painted the same color and are 1m apart.

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