

NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 2

Fall 2021

Problem 2.

A number is said to be “good” if it has seven digits, it is a multiple of 388, and it has 388 as the last three digits. How many good numbers are there?

Solution.

Answer: There are 93 good numbers.

The number $\overline{a_1a_2a_3a_4388}$ is divisible by 388, if and only if, $\overline{a_1a_2a_3a_4000} = \overline{a_1a_2a_3a_4} \times 1000$ is.

Now, we have that $388 = 97 \cdot 2^2$ is the prime factorization of 388 and $\gcd(1000, 388) = 4$. Then, $\overline{a_1a_2a_3a_4} \times 1000$ is divisible by 388, if and only if, $\overline{a_1a_2a_3a_4}$ is divisible by 97.

The first and last number $\overline{a_1a_2a_3a_4}$ with $a_1 \neq 0$ divisible by 97 are

$$1067 = 97 \times 11 \quad \text{and} \quad 9991 = 97 \times 103.$$

Thus, there are $103 - 11 + 1 = 93$ good numbers.