

NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 5

Fall 2021

Problem 5.

The points in the Cartesian plane are colored with one of the colors yellow, blue, or red. Every point can only be colored by one color. Prove that there are two points in the plane of the same color that are distance 1 from each other.

Solution.

Let A be any point in the plane and assume it is colored yellow. Let B be a point at a distance $\sqrt{3}$ from A that is colored with a different color (say blue). If such B does not exist, there is a circle of radius $\sqrt{3}$ centered at A such that every point in the circle is yellow, and then there must be two points in the circle at a distance 1 from each other.

Let C and D be two points at opposite sides of the line AB such that AC , AD , BC , and BD have length 1. Therefore, C and D must be colored red. Now, notice that CD is the perpendicular bisector of AB . It follows by the Pythagorean theorem that CD has length one, finishing the proof.