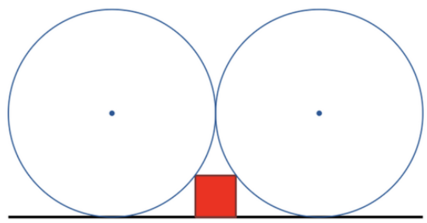
NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 7

Fall 2025

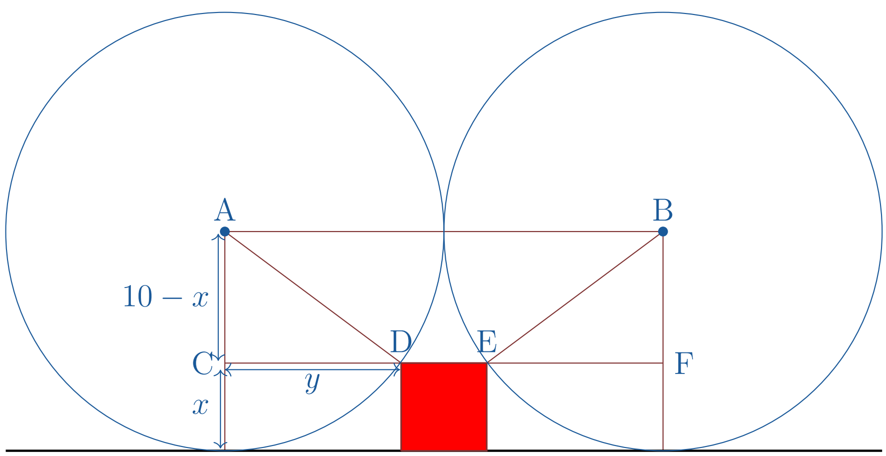
Problem 7

Find the area of the red square placed between the two circles of radius 10 meters in the diagram:



Justify your answer.

Solution. Let x be the side length of the red square and y be the length of CD in the diagram



By symmetry, $|\overline{\mathrm{EF}}| = |\overline{\mathrm{CD}}| = y$. Since, the length of segment $\overline{\mathrm{AB}}$ must equal the length of $\overline{\mathrm{CF}}$, we get

$$20 = |\overline{\mathrm{AB}}| = |\overline{\mathrm{CF}}| = |\overline{\mathrm{CD}}| + |\overline{\mathrm{DE}}| + |\overline{\mathrm{EF}}| = 2y + x,$$

or equivalently

$$y = (20 - x)/2 = 10 - \frac{x}{2}. \tag{1}$$

To obtain another relationship between x and y, we consider the triangle ΔACD, which is right angled at C with sides $|\overline{\mathrm{AC}}| = 10 - x$, $|\overline{\mathrm{CD}}| = y$ and $|\overline{\mathrm{AD}}| = 10$. Using the Pythagorus theorem, we get

$$(10 - x)^2 + y^2 = 10^2.$$

Using the expression from (1) in the above equation, we get

$$\begin{aligned}
(10-x)^2 + (10-\tfrac{x}{2})^2 &= 10^2\\
\Rightarrow 100 - 20x + x^2 + 100 - 10x + \tfrac{x^2}{4} &= 100\\
\Rightarrow \tfrac{5x^2}{4} - 30x + 100 &= 0\\
\Rightarrow 5x^2 - 120x + 400 &= 0\\
\Rightarrow x^2 - 24x + 80 &= 0\\
\Rightarrow (x-4)(x-20) &= 0.
\end{aligned}$$

Thus $x = 4$ or $x = 20$. Since $|\overline{\mathrm{DE}}| = x$ is strictly less than $|\overline{\mathrm{AB}}| = 20$, x cannot equal 20. Thus, x must equal 4. Therefore,

$$\text{Area of the red square} = x^2 = 16 \text{ m}^2.$$