NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 3

Spring 2026

Problem 3

For every integer n, is it true that 3 divides at least one of the integers n, $n+2$, or $n+4$? Provide a proof if the statement is true, or a counterexample if it is false.

Solution. Yes, it is true, and one can reason as follows. Let n be an arbitrary integer. If 3 divides n, there is nothing to prove. Otherwise, upon dividing n by 3, the remainder is 1 or 2. If the remainder is 1, then $n = 3p + 1$ for some integer p. Therefore,

$$n + 2 = (3p + 1) + 2 = 3p + 3 = 3(p + 1).$$

Thus, 3 divides $n + 2$. On the other hand, if the remainder is 2, then $n = 3q + 2$ for some integer q. Therefore,

$$n + 4 = (3q + 2) + 4 = 3q + 6 = 3(q + 2).$$

Thus, 3 divides $n + 4$. Consequently, 3 divides at least one of the integers n, $n + 2$, or $n + 4$.