NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 4

Spring 2026

Problem 4

An automated teller machine (ATM) is stocked exclusively with \$20 and \$50 bills. Assuming the machine has an unlimited supply of each, determine all possible total amounts the machine can dispense. Justify your answer.

Solution. We claim that the ATM can dispense $\$(10n)$ bills, where $n = 2$ or $n \geq 4$. Firstly, since the ATM can dispense one \$20 bill, the result is clear for $n = 2$. To prove that it is true for each $n \geq 4$, we use the principle of mathematical induction.

Base Case: If $n = 4$, then the result is clear since the ATM can dispense two \$20 bills.

Induction Step: Suppose that the ATM can dispense $\$(10n)$ bills. We show that it can dispense $\$10(n + 1)$ bills. There are two cases to consider.

Case 1. First suppose that in dispensing $\$(10n)$ bills, at least one \$50 bill was used. Then $10n = 20m + 50k$, where $m \geq 0$ and $k \geq 1$. Therefore,

$$10(n + 1) = 10n + 10 = (20m + 50k) + 10 = 20m + 50(k - 1) + 60 = 20(m + 3) + 50(k - 1).$$

Thus, in this case, the ATM can dispense $\$10(n + 1)$ bills.

Case 2. Next suppose that in dispensing $\$(10n)$ bills, no \$50 bill was used. Then at least two \$20 bills must have been used since $n \geq 4$, Therefore, $10n = 20m$, where $m \geq 2$. Thus,

$$10(n + 1) = 10n + 10 = 20m + 10 = 18m + 50.$$

It follows that the ATM can dispense $\$10(n + 1)$ bills in this case too.

We conclude by the principle of mathematical induction that the ATM can dispense $\$(10n)$ bills for each $n \geq 4$.