NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 1

Spring 2026

Problem 1

We call a year **Harmonious** if the sum of its digits is a divisor of the year itself. For example, the year 2025 was Harmonious because $2+0+2+5 = 9$, and 2025 is divisible by 9. Whereas, 2026, is clearly not Harmonious as $2+0+2+6 = 10$ does not divide 2026. What are the next three Harmonious years after 2026? Justify your answer.

Solution. We have:

- $2+0+2+7 = 11$ and 11 does not divide 2027, so 2027 is not Harmonious.
- $2+0+2+8 = 12$ and 12 divides 2027, so 2028 **is** Harmonious.
- $2+0+2+9 = 13$ and 13 does not divide 2029, so 2029 is not Harmonious.
- $2+0+3+0 = 5$ and 5 divides 2030, so 2030 **is** Harmonious.
- $2+0+3+1 = 6$ and 6 does not divide 2031, so 2031 is not Harmonious.
- $2+0+3+2 = 7$ and 7 does not divide 2032, so 2032 is not Harmonious.
- $2+0+3+3 = 8$ and 8 does not divide 2032, so 2032 is not Harmonious.
- $2+0+3+4 = 9$ and 9 divides 2034, so 2034 **is** Harmonious.

Thus, the next three Harmonious years after 2026 are: **2028**, **2030**, and **2034**.