

# Why Optimization Is Faster than Solving Systems of Equations: A Qualitative Explanation

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**Formulation of the problem.** At first glance, these two problems can be reduced to each other, so their computational complexity should be comparable:

- optimization  $f \rightarrow \min$  can be reduced to solving a system of equations obtained by equating all partial derivatives to 0:  $\frac{\partial f}{\partial x_i} = 0$ ; and
- solving a system of equations  $f_1(x_1, \dots, x_n) = 0, \dots, f_n(x_1, \dots, x_n) = 0$  is equivalent to minimizing the sum  $\sum_{i=1}^n (f_i(x_1, \dots, x_n))^2$ .

However, empirically, optimization is faster; see, e.g., [1]. How can we explain this?

**Possible explanation.** In general, the more inputs we have, the more computation time the problem requires. For both optimization problem and the problem of solving a system of equation, the inputs are functions. For our two problems:

- To describe an optimization problem, we need to describe only one function  $f(x_1, \dots, x_n)$  – which is minimized (or maximized).
- On the other hand, to describe a system of  $n$  equations with  $n$  unknowns, we need to describe  $n$  functions  $f_1(x_1, \dots, x_n), \dots, f_n(x_1, \dots, x_n)$ .

So, not surprisingly, optimization problems are, in general, faster to solve.

## References

- [1] R. L. Muhanna, R. L. Mullen, and M. V. Rama Rao, “Nonlinear Interval Finite Elements for Beams”, *Proceedings of the Second International Conference on Vulnerability and Risk Analysis and Management (ICVRAM) and the Sixth International Symposium on Uncertainty, Modeling, and Analysis (ISUMA)*, Liverpool, UK, July 13–16, 2014.