## ERGODIC THEOREMS FOR $\phi$ -NONEXPANSIVE SEQUENCES IN BANACH SPACES

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ABSTRACT. Let X be a real Banach space with norm  $|\cdot|$ . A sequence  $\{x_n\}$  in X is called a nonexpansive sequence if

 $|x_{i+1} - x_{j+1}| \le |x_i - x_j|, \quad \forall i, j \ge 1.$ 

When X is a real Hilbert space H, ergodic theorems for such sequences were proved by Djafari Rouhani in 1981, and it was stated as an open problem whether those results could be extended to Banach spaces. By introducing the notion of  $\phi$ -nonexpansive sequences and mappings in X, we give a partial affirmative answer to this open question.

**Keywords:** Asymptotic behavior; asymptotically regular sequence; strong convergence; weak convergence;  $\phi$ -nonexpansive sequence.

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