An Arbitrary Preference Relation Can Be Represented in Qualitative Choice Logic: A Remark

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Formulation of the problem. How do we describe what we want? Out of several properties $P_1, \ldots, P_n$ of the desired alternatives, we may want to have some properties and no others. For example, when we are looking for a house, we may want either a big house (P_1) or a smaller house ($\neg P_1$) in a good school district (P_2), but not in a bad neighborhood ($\neg P_3$). In such situations, what we want can be described by a propositional formula; for example, in the above case, this formula takes the form $(P_1 \vee (\neg P_1 \,\&\, P_2)) \,\&\, \neg P_3$.

But what if such an ideal object is not available? To make a decision in such cases, we need to also describe preferences: e.g., we can say "we want A but if A is not possible, then we should have B." This condition is described by $A \vec{\times} B$. For example, if we want to nominate a student for the best student award, we may want to have a straight-A student but if there no such students, at least a student with high GPA. The idea of adding the new connective $\vec{\times}$ to propositional logic first appeared in [2]; the resulting logic is called *Qualitative Choice Logic*.

Several other additional connectives have been proposed to describe preferences; see, e.g., [1]. These additional connectives help to speed up computations. A natural question is: are these other connectives needed to represent human preferences – or, in principle, $\vec{\times}$ is sufficient?

Our answer. In this talk, we provide a positive answer to this question: yes, every preference relation can be represented in Qualitative Choice Logic.

Indeed, with n properties, we can, in principle, have 2^n possible situations described by formulas $P(\varepsilon)$ of the type $P_1^{\varepsilon_1} \,\&\, \ldots \,\&\, P_n^{\varepsilon_n}$, where $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$, $\varepsilon_i \in \{-, +\}$, P_i^+ means P_i, and P_i^- means $\neg P_i$. A general preference relation is a strict partial order $<$ between such formulas, where $a < b$ means that if both a and b are available, we prefer b.

To describe this relation in Qualitative Choice Logic, we take all the formulas $b \vec{\times} a$ corresponding to pairs (a, b) for which $a < b$. One can see that this indeed leads to the desired representation.

References

[1] M. Bernreiter, J. Maly, and S. Woltran, "Choice logics and their computational properties", *Artificial Intelligence*, 2022, Vol. 311, Paper 103755.

[2] G. Brewka, S. Benferhat, and D. Le Berre, "Qualitative Choice Logic", *Artificial Intelligence*, 2004, Vol. 157, pp. 203–237.