

# Why Earthquake Statistics Vary with Fault Size: An Invariance-Based Qualitative Explanations

Jannatun N. Jotey and Vladik Kreinovich  
Department of Computer Science, University of Texas at El Paso  
jnjotey@miners.utep.edu vladik@utep.edu

**Formulation of the problem.** Earthquakes can be devastating, so researchers have been studying them starting with ancient times. The main emphasis have always been on areas where the strongest, the most devastating earthquakes are possible. According to modern geosciences, earthquakes are mainly occurring in the close vicinity of faults, i.e., places where there is discontinuity – between tectonic plates, between terranes, etc. In general, the larger the fault, the more potential energy it contains, so the stronger the earthquakes.

Usually, a strong earthquake is followed by a sequence of weaker earthquakes whose strength  $s(t)$  decreases with time  $t$  as a power law  $s(t) \approx C \cdot t^{-a}$  for some  $C$  and  $a$ . With more accurate measuring instruments, it is now possible to study smaller-size earthquakes – that correspond to smaller-size faults – as well. Researchers expected that the resulting sequences of aftershocks would follow a similar power law, but surprisingly, it turned out that for such faults, the strength of the follow-up earthquakes does not decrease with time at all.

Recent research provides an explanation based on detailed geophysical model. In this talk, we show that – at least on the qualitative level – this phenomenon can be explained based on general invariance ideas.

**Our explanation.** Numerical values of physical quantities depend on the choice of the measuring units and on the choice of the starting point. In many cases, there is no preferred measuring unit, so it makes sense to conclude that the dependence  $y = f(x)$  between the quantities should not depend on what unit we select for measuring these quantities.

When we change a measuring unit to a new one which is  $\lambda$  times smaller, all the numerical values are multiplied by  $\lambda$ :  $x \rightarrow x' = \lambda \cdot x$ ; for example, 2 m becomes  $2 \cdot 100 = 200$  cm. Of course, when we change a measuring unit for  $x$ , we need to appropriately change a measuring unit for  $y$ . For example, the formula  $y = x^3$  for the volume of a cube does not depend on the units, but when we change the measuring unit for the cube's linear size  $x$ , we need to appropriately change the unit for measuring the volume  $y$ : from  $\text{m}^3$  to  $\text{cm}^3$ . In general, for every  $\lambda > 0$ , there exists a  $\mu > 0$  (depending on  $\lambda$ ) for which, once we have  $y = f(x)$ , we should also have  $y' = f(x')$ , where  $x' = \lambda \cdot x$  and  $y' = \mu(\lambda) \cdot y$ . It is known that the only measurable functions  $f(x)$  satisfying this property are power laws – this explains the larger-faults power law.

While for strength and for many other physical quantities, there is a natural starting point – e.g., 0 for earthquake strength – for some other quantities like time, there is no natural starting point. For aftershocks following a strong earthquake, there is a natural starting point for time – the time of this strong earthquake. For larger-size faults, a typical strong earthquake drastically changes the geological structure, releases the stress, so there is a clear difference between the situations before and after the major earthquake – and the time of this earthquake thus serves as a natural starting point for measuring time. In contrast, small earthquakes (typical for smaller faults) do not have enough power to make drastic changes, there is almost no difference in the geological structure before and after the earthquake, and thus, there is no natural starting point for time. If we change a starting point to a new ones  $t_0$  moments in the past, then each numerical value  $t$  is replaced by the new value  $t' = t + t_0$ . In this case, the additional requirement that the physics should not depend on the selection of the starting point means that for every  $t_0$ , we should have an appropriate re-scaling  $s \rightarrow s' = \mu(t_0) \cdot s$  for which  $s = s(t)$  would imply  $s' = s(t')$ . Since we know that  $s(t) = C \cdot t^a$ , this condition implies that  $a = 0$ , i.e., that  $s(t)$  is a constant – this is exactly what we observe.