How to Explain Empirical Metric on the Set of Colors

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Formulation of the problem. It is known that human color perception corresponds to the 3D space – in the sense that every color that we see can be perfectly emulated by a combination of three colors. Researchers are also interested in how we perceive the difference between different colors. For this purpose, they use volunteers to estimate the distance between different colors by a number. There are formulas that allow us to predict, for every two close colors, the user's estimate of the distance between these two colors.

It is desirable, based on these formula, to be able to predict the subjective distance between any two colors – which are not necessarily close to each other. If this was the geometric distance – e.g., distance between two locations on Earth or two locations in space – this would be straightforward to do: for each path between the two colors, we can find the total length of this path (by adding the lengths of all it short segments that form this path), and then we can define the distance $d(a,b)$ between the two points a and b as the shortest length of the path that connects these two points. We can perform the same procedure for two colors a and b and get the length $d(a,b)$ of the shortest path that connects a and b. However, in contrast to the geometric distance, the resulting value $d(a,b)$ is different from the estimate $e(a,b)$ provided by humans: namely, $e(a,b) \approx C \cdot \ln(d(a,b))$; see, e.g., [1]. How can we explain this empirical formula?

Our explanation. On a ruler, the difference between two values is proportional to the number of different readings separating these values. For example, in a metric ruler, where we have readings at a millimeter distance, there are 20 readings between 1 cm and 3 cm. We can similarly describe our perception: for each value x_0 of a quantity, values x that are very close to x_0 cannot be distinguished from x_0. As we increase x, we will come up with the smallest value $x_1 > x_0$ that is distinguishable from x_0. Then, we will similarly have the smallest value $x_2 > x_1$ that is distinguishable from x_1, etc. If we start with some fixed value x_0, then a natural way for us to gauge a value $x > x_0$ is by the number n of such distinguishable values x_i between x_0 and x.

To find the perceived distance n as a function of the actual distance x, let us analyze what will be, for each value x, the smallest value $y = f(x) > x$ which is distinguishable from x. There is no preferred measuring unit for distance, so it makes sense to require that the relation $y = f(x)$ remain the same if we change the unit to a new one which is λ times smaller, i.e., if we replace x and y with $x' = \lambda \cdot x$ and $y' = \lambda \cdot y$. So, $y = f(x)$ implies that $f(\lambda \cdot x) = \lambda \cdot f(x)$. In particular, for $x = 1$, we get $f(\lambda) = c \cdot \lambda$, where $c = f(1)$. Thus, $x_1 = c \cdot x_0$, $x_2 = c \cdot x_1 = c^2 \cdot x_0$, and, in general, $x = c^n \cdot x_0$, so $n = \log_c(x/x_0)$. This explains why the perceived distance n is proportional to the logarithm of the actual distance.

References

[1] R. Bujack, E. Teti, J. Miller, E. Caffrey, and T. L. Turton, "The non-Riemannian nature of perceptual color space", *Proceedings of the National Academy of Science of the USA*, 2022, Vol. 119, No. 18, Paper e2119753119, https://doi.org/10.1073/pnas.2119753119.