

Modal Logics of Horn Closures of the Random Kripke Frame

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Almost sure validities in random graphs. There are well-known connections between the probability and the logic. The majority of well-known results in this area concern the first-order relational language. The Zero-one law for first-order logic, proved independently in [1] and [2], is likely the most celebrated of these results.

Other important results include descriptions of the sentences that are true with a probability that tends to one, or *asymptotically almost surely*. Gaifman [3] gives an axiomatization for almost sure truths in Radó graph, a model of countable random graph, in the first-order language.

It is of interest to address the problem of almost sure truths in the modal language. With Kripke semantics, we interpret modal formulas as properties of directed graphs, which we call Kripke frames. The paper [4] provides a complete and sound axiomatization for the theory of almost sure validities in a countable Kripke frame, and [5] uses this result to identify some of the almost sure validities in the finite frames. The problem of complete axiomatization for the finite frames is still open.

Another open question is about the almost sure validities in a given class \mathcal{F} of Kripke frames. In this paper we develop an approach to study the random Kripke frames in Horn classes. We call a class of Kripke frames a Horn class if it is defined by conditions of the form

$$\forall x_1, \dots, x_n (x_{i_1} R x_{j_1} \wedge \dots \wedge x_{i_k} R x_{j_k} \rightarrow x_m R x_l).$$

If a modal logic L defines a Horn class, then for any Kripke frame $F = (X, R)$ there exists the minimal frame F^L , called the L -closure of F , such that $F^L \models L$ and the relation of F^L extends R .

Results. We develop axiomatizations for the modal theories of almost sure validities in the L -closures, where L is given by combinations of axioms of reflexivity, symmetry, transitivity, and the Euclidean property $xRy \wedge xRz \rightarrow yRz$.

We established that for any modal logic L that defines a Horn class of transitive or Euclidean frames, the modal logic of almost sure validities in the L -closures is the well-known system **S5**, which is the logic of equivalence relations.

We also showed that for any modal logic L , the logic of almost sure validities in the L -closures extends both L and \mathbf{ML}^r , the logic of the countable random frame introduced in [4]. Using this result, we established lower bounds on the logics of almost sure validities in the classes of reflexive and symmetric frames.

References

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