Criteria of local tabularity of products of modal logics

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A logic is *locally tabular*, if each of its finite-variable fragments contains only a finite number of pairwise nonequivalent formulas. It is well known that for unimodal transitive logics, local tabularity is equivalent to finite height [1], [2]. In the non-transitive unimodal, and in the polymodal case, no axiomatic criterion of local tabularity is known.

We study locally tabular *products of modal logics*. For frames $F = (X, R)$ and $G = (Y, S)$, the *product frame* $F \times G$ is the frame $(X \times Y,\, R^\times, S^\times)$, where

$$R^\times = \{((a,\, b),\, (a',\, b)) \mid b \in Y,\, aRa'\}; \quad S^\times = \{((a,\, b),\, (a,\, b')) \mid a \in X,\, bSb'\}.$$

The *product* $L_1 \times L_2$ *of logics* L_1, L_2 is the bimodal logic of the class of products

$$\{F \times G \mid F \models L_1,\, G \models L_2\}.$$

Local tabularity is established for some families of products. The products with the logic of equivalence relations S5 provide a valuable example: while S5×S5 lacks local tabularity [3], it was shown in [4] that every extension of S5 × S5 is locally tabular. A family of locally tabular modal products was identified in [5]. For close systems, *intuitionistic modal logics*, locally tabular families were identified in [6], [7], [8], and a recent manuscript [9].

In the product $L_1 \times L_2$ of two Kripke complete consistent logics, local tabularity of L_1 and L_2 is necessary for local tabularity of $L_1 \times L_2$. However, the product of two locally tabular logics can be not locally tabular. The simplest example is the logic S5 × S5 [3] . We provide extra semantic and axiomatic conditions which give criteria of local tabularity of the product of two locally tabular logics: bounded cluster property of one of the factors; a condition we call *product reducible path property*; finiteness of the one-variable fragment of the product. Then we apply them to identify new families of locally tabular products.

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References

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