Reversible and quantum computing involving random processes: local time naturally appears

Fernando De Santiago, Nicole Favela, Christian Garcia,
Dimitri Lyon, Emilia Rivas, and Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso
500 W. University, El Paso, TX 79968, USA
fdesantiago@miners.utep.edu;nefavela@miners.utep.edu, cgarcia83@miners.utep.edu,
delyon@miners.utep.edu, erivas6@miners.utep.edu, vladik@utep.edu

Reversible computing: why. One of the big problems of modern computing is that computers use a large amount of energy, up to 10% of all the world's energy consumption – and this portion increases. There are technological reasons for this consumption, but there is also a fundamental physical reason: according to thermodynamics, irreversible processes increase entropy and thus, cause heat release. And computing uses a lot of irreversible processes: e.g., an and-gate transforms two signals a and b into a single signal $c = a \,\&\, b$. When $c = 0$, we cannot uniquely reconstruct the inputs (a, b); it could be $(0, 0)$, or $(0, 1)$, or $(1, 0)$. To decrease computers' energy consumption (and heating the environment), it is desirable to make computations reversible.

Limitation to reversible computing is also needed for quantum computing, where computation is done on the level of elementary particles – and on this level, all physical processes are reversible.

Reversible computing: how. To make computations reversible, a natural idea is to add auxiliary inputs. For example, we can make an and-gate reversible if we add an auxiliary input y and transform (a, b, y) into $(a, b, y \oplus (a \,\&\, b))$, where $\oplus$ denoted exclusive "or" (i.e., addition modulo 2). Similarly, to make computations $x \to f(x)$ with real numbers reversible, we can add additional input y and transform (x, y) into $(f(x), g(x, y))$ for some $g(x, y)$. Here, reversible means that the number of output states be equal to the number of input states. If we compute with accuracy ε, then the number of input states is proportional to the area of the (x, y) domain. In these terms, reversible means area-preserving, and this leads to $g(x, y) = y/f'(x)$, where $f'(x)$ means the derivative.

What if we want to simulate a random process? Many real-life processes are random. It is therefore important to simulate them. In these simulations, just like in all other simulations, it is desirable to use reversible and quantum computing. For this purpose, we need to replace possibly irreversible computations $t \mapsto x(t)$ with reversible ones $(t, y) \mapsto (x(t), g(t, y))$ for some function $g(t, y)$. For smooth processes, as we have mentioned, we should take $g(t, y) = y/x'(t)$. So, we need to extend this expression to general – not necessarily smooth – processes.

Interestingly, such an extension, in effect, already exists. To be more precise, $1/x'(t)$ is a smooth case of *local time* $\ell(t)$ which is defined as follows. For every x-interval $I = [x, x + \varepsilon]$, we consider the overall duration $d(\varepsilon)$ of all the time intervals at which we had $x(t) \in I$. Then, we define $\ell(t)$ as the limit of the ratio $d(\varepsilon)/\varepsilon$ when $x = x(t)$. It is known that in the smooth case, $\ell(t) = 1/x'(t)$, and that in the general case, when we have a sequence of smooth processes approximating a given one, the resulting values $1/x'(t)$ tend to $\ell(t)$. Thus, in general, we can use local time to have a reversible or quantum simulation of a random process $x(t)$ as $(t, y) \mapsto (x(t), y \cdot \ell(t))$.