Why Precision, Recall, and Accuracy – and Not Some Other Characteristics?

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Formulation of the problem. Classification methods are often not perfect. In addition to true positive (TP) and true negative (TN) cases, we also have false positive (FP) and false negative (FN) cases. To gauge the quality of a classification method, we need to take into account the numbers of all these four categories. In principle, we can have many different combinations of these four numbers. Empirically, the following three combinations are most frequently used: precision $P = TP/(TP + FP)$, recall $R = TP/(TP + FN)$, and accuracy $A = (TP+TN)/(TP+TN+FP+FN)$ – and well as their combination $F1 = P \cdot R/(P+R)$. A natural question is: why these characteristics and not others?

Towards an explanation. Each correct classification brings benefits, each false classification brings losses. The method should be applied if the benefits are larger than the losses. With respect to benefits, sometimes, benefits b_{TP} and b_{TN} of TP and TN are similar, and sometimes, one of them brings more benefits: e.g., detecting cancer may save a life, while correctly identifying a non-cancerous tumor simply saves a patient from a few further procedures. In principle, we could have three cases: $b_{TP} \approx b_{TN}$, $b_{TP} \gg b_{TN}$, and $b_{TN} \gg b_{TP}$. If TN brings more benefits, we can simply rename negative to positive, so we have two cases: $b_{TP} \approx b_{TN}$ and $b_{TP} \gg b_{TN}$. In the first approximation, when we ignore small numbers and small differences, the first case means $b_{TP} = b_{TN}$, and the second means $b_{TP} > 0$ and $b_{TN} = 0$. Similarly for losses, in the first approximation, we can consider three possible cases: the case when $\ell_{FP} = \ell_{FN}$, the case when $\ell_{FP} > 0$ and $\ell_{FN} = 0$, and the case when $\ell_{FN} > 0$ and $\ell_{FP} = 0$.

Let us first consider the case when $b_{TP} > 0$, $b_{TN} = 0$, $\ell_{FP} > 0$ and $\ell_{FN} = 0$. In this case, the method is beneficial if $b_{TP} \cdot TP > \ell_{FP} \cdot FP$, i.e., equivalently, when $r_1 \stackrel{\text{def}}{=} TP/FP > \ell_{FP}/b_{TP}$. The larger the ratio r_1, the more cases when this method is useful. So, the quality of the method is larger if the ratio r_1 is larger. Alternatively, we can take any strictly increasing function of r_1, e.g., $1/(1 + 1/r_1)$. Applying this function to $r_1 = TP/FP$, we get exactly the precision – which explains why precision is used.

In the case when $b_{TP} > 0$, $b_{TN} = 0$, $\ell_{FN} > 0$, and $\ell_{FP} = 0$, a similar argument leads to recall. In the case when $b_{TP} = b_{TN}$ and $\ell_{FP} = \ell_{FN}$, a similar argument leads to accuracy.

Why only three? For general values of benefits and losses, the method is effective if $b_{TP} \cdot TP + b_{TN} \cdot TN > \ell_{FP} \cdot FP + \ell_{FN} \cdot FN$. If we divide both sides by TP, we get an equivalent inequality $b_{TP} + b_{TN} \cdot TN/TP > \ell_{FP} \cdot FP/TP + \ell_{FN} \cdot FN/TP$ with three unknown ratios $R_1 \stackrel{\text{def}}{=} FP/TP$, $R_2 \stackrel{\text{def}}{=} FN/TP$, and $R_3 \stackrel{\text{def}}{=} TN/TP$. One can check that, by dividing both the numerator and the denominator of the expressions for P, R, and A by TP, that these three values depend only on these three ratios: $P = 1/(1 + R_1)$, $R = 1/(1 + R_2)$, and $A = (1 + R_3)/(1 + R_1 + R_2 + R_3)$. Thus, when we know the values of P, R, and A, we have 3 equations from which we can determine all three unknown ratios: $R_1 = 1/P - 1$, $R_2 = 1/A - 1$, and $R_3 = (A \cdot (R_1 + R_2 + 1) - 1)/(1 - A)$. Hence, once we know P, R, and A, we will be able to predict, for each combination of benefits and losses, whether this method is applicable. So, the three characteristics are indeed sufficient – all other characteristics can be described in terms of these three, just like F1 can be.