

Asymptotic behavior of the normalized time changed Wiener process

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Abstract

The time changed Wiener process is a stochastic process of the form $X_t = B_{\tau_t}$, where a standard Wiener process B is evaluated at a random, adapted, and non-decreasing time scale τ_t . This is often used to model stochastic volatility, default times and credit spreads, interspike intervals in neuronal activity, etc. In this work, we investigate the asymptotic behavior of a normalized time-changed Wiener process obtained by composing Brownian motion with the inverse of a diffusion process. Such time changes destroy the independent increment properties of Brownian motion, making classical functional limit theorems nontrivial to apply and studying the weak convergence of the sequence. Under mild assumptions on the underlying diffusion, we derive that the law of large numbers remains valid for this class of time-changed Wiener processes. Our analysis relies on martingale techniques, properties of inverse processes, and the normalization of the time changed Wiener process.