On Local Finiteness of Modal K4 Algebras

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An algebra $\mathbb{A}$ is said to be locally finite if every finitely generated subalgebra of $\mathbb{A}$ is finite. The (equational) logic of a locally finite algebra has the finite model property, and hence is an attractive attribute for algebras to have. We provide a sufficient condition for local finiteness in modal $K4$ algebras, obtained by identifying a structure which is necessarily present in non-locally finite modal $K4$ algebras. This condition becomes both necessary and sufficient when considered in complete modal $K4$ algebras. Next, we translate this condition into a pair of order-theoretic conditions on transitive Kripke frames, providing a classification of local finiteness on their dual modal algebras. We further show that the logic of any class of well-founded transitive relations with no infinite antichains has the finite model property, and conclude that the logic of the class of well-quasi-orderings (well-founded quasi-orders that have no infinite antichains) has the finite model property.