A Formal Framework for Rule-Based Triage with Priority Queueing Dynamics

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***Abstract*—Triage systems are critical for prioritizing patients in emergency settings, yet their decision rules are often informally specified and analyzed in isolation from system-level constraints. In this work, we present a formal mathematical framework for a rule-based triage algorithm and integrate it with a priority queueing model to capture patient flow dynamics. The proposed formulation enables rigorous reasoning about correctness, determinism, and operational properties such as waiting time and prioritization fairness. This framework provides a foundation for future extensions involving uncertainty and optimization.**

I. Introduction

Triage is the process of prioritizing patients based on the urgency of their clinical condition in settings with limited medical resources, such as emergency departments. These protocols aim to ensure that patients requiring immediate intervention are identified and treated promptly, while lower priority cases are safely deferred.

Despite their widespread use, triage systems are typically specified as rule-based procedures, which limits their formal analysis and integration with system-level models of patient flow. The triage rules considered in this work are based on an existing protocol used in practice, which we reinterpret through a formal mathematical framework.

In this work, we formalize and analyze a rule-based triage protocol currently used in clinical practice. Specifically, we:

- Provide a mathematical formulation of the triage decision process as a deterministic classifier.
- Characterize the induced partition of the patient state space.
- Integrate the classifier with a priority queueing model to analyze patient scheduling.

This combined framework enables a principled analysis of triage behavior under resource constraints.

II. Triage Model

A. Problem Definition

We consider the problem of assigning a triage category to a patient based on their clinical state at arrival, using an existing rule-based triage protocol. The goal is to formalize this decision process as a deterministic mapping from patient features to urgency levels.

B. Patient State Space

We define the patient state space as

$$\mathcal{X} := \{0,1\} \times \{0,1\} \times \mathbb{N}_0 \times \mathbb{R}^5.$$

An individual patient is represented by a state vector

$$x = (R, H, D, HR, BP, RR, Temp, SpO_2) \in \mathcal{X},$$

where:

- $R \in \{0,1\}$ indicates whether the patient requires immediate resuscitation,
- $H \in \{0,1\}$ indicates whether the patient is high-risk and should not wait,
- $D \in \mathbb{N}_0$ denotes the number of required diagnostic or therapeutic actions,
- $HR, BP, RR, Temp, SpO_2 \in \mathbb{R}$ correspond to vital signs.

C. Output Space

We define the set of triage categories as

$$\mathcal{C} = \{\text{Red}, \text{Orange}, \text{Yellow}, \text{Green}, \text{Blue}\}.$$

D. Vital Instability Predicate

We define a predicate $V : \mathcal{X} \to \{0,1\}$ capturing physiological instability:

$$\begin{aligned} V(x) = 1 \iff\ & HR > 150 \\ & \lor (BP > 170 \lor BP < 100) \\ & \lor (RR < 15 \lor RR > 30) \\ & \lor (Temp < 30 \lor Temp > 38.5) \\ & \lor SpO_2 < 85 \end{aligned} \tag{1}$$

E. Triage Classifier

We define a deterministic classifier

$$T : \mathcal{X} \to \mathcal{C}$$

that assigns a triage category to each patient state.

The classification rules are:

$$T(x) = \begin{cases} \text{Red} & R = 1 \\ \text{Orange} & R = 0 \wedge H = 1 \\ \text{Blue} & R = 0 \wedge H = 0 \wedge D = 0 \\ \text{Green} & R = 0 \wedge H = 0 \wedge D = 1 \\ \text{Orange} & R = 0 \wedge H = 0 \wedge D > 1 \wedge V(x) = 1 \\ \text{Yellow} & \text{otherwise} \end{cases} \tag{2}$$

F. Logical Structure

The classifier T induces a partition of the patient state space into disjoint regions:

$$\mathcal{X} = \bigsqcup_{c \in \mathcal{C}} \mathcal{X}_c, \quad \mathcal{X}_c = \{x \in \mathcal{X} : T(x) = c\}.$$

Each subset $\mathcal{X}_c$ contains all patient states that are assigned to the same triage category c.

For example:

$$\mathcal{X}_{\text{Red}} = \{x \in \mathcal{X} : R = 1\}, \tag{3}$$

$$\mathcal{X}_{\text{Orange}} = \{R = 0, H = 1\} \cup \{R = 0, H = 0, D > 1, V(x) = 1\}. \tag{4}$$

These regions are mutually disjoint, meaning that no patient state belongs to more than one category, and exhaustive, meaning that every possible patient state belongs to exactly one category.

Therefore, the classifier T defines a total and deterministic function, assigning a unique triage level to every patient.

G. Structural Induction Properties

The classifier T can be interpreted as a finite-depth decision tree. Its correctness follows from the ordered evaluation of conditions, where higher-priority rules dominate lower ones.

In particular:

$$R = 1 \Rightarrow T(x) = \text{Red}$$

ensuring that critical conditions override all subsequent rules.

This establishes a precedence structure over decision variables.

III. Queueing Model

A. Priority Mapping

Define a priority function:

$$p : \mathcal{C} \to \mathbb{N}$$

$$p : \mathcal{C} \to \mathbb{N}, \quad \begin{cases} \text{Red} \mapsto 1 \\ \text{Orange} \mapsto 2 \\ \text{Yellow} \mapsto 3 \\ \text{Green} \mapsto 4 \\ \text{Blue} \mapsto 5 \end{cases} \tag{5}$$

B. System Dynamics

Patients arrive over time and form a queue:

$$Q(t) = \{x_i \text{ waiting at time } t\}.$$

Each patient is assigned a priority $p_i = p(T(x_i))$.

C. Scheduling Rule

We define the selection rule:

$$S(Q(t)) = \arg \min_{x_i \in Q(t)} p(T(x_i)).$$

The scheduler S selects the next patient to be treated by minimizing the assigned priority value. Since lower values correspond to higher urgency, this implements a strict priority discipline in which more critical patients are always served first. In cases where multiple patients share the same priority level, a first-in, first-out (FIFO) policy is applied.

This defines a non-preemptive priority queue.

IV. Properties

A. Determinism

Since T is deterministic, the scheduling rule S is also deterministic.

B. Priority Consistency

If $p_i < p_j$, then patient x_i is served before x_j.

C. Computational Complexity

The triage function requires a constant number of operations:

$$\mathcal{O}(1)$$

Thus, classification and scheduling decisions are computationally efficient.

D. System Behavior

Under high load, lower-priority patients may experience extended waiting times. This highlights a trade-off between urgency prioritization and fairness.

V. Conclusion

The proposed framework provides a unified view of triage classification and patient flow. By formalizing both components, we enable analysis of correctness and system-level behavior.

Future work includes incorporating uncertainty in patient measurements and analyzing robustness of classification decisions.