Local linear stability analysis of dual-pairing summation-by-parts methods for nonlinear conservation laws

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Abstract

A recent study by Gassner et al. [J. Sci. Comput. 90:79 (2022)] demonstrates that local linear stability—that is, ensuring the asymptotic numerical growth rate does not exceed the continuous growth rate—is crucial for achieving accurate numerical simulations of nonlinear conservation laws. While nonlinear entropy stability is necessary for numerical stability (i.e., ensuring the boundedness of nonlinear numerical solutions), local linear stability is essential to prevent unresolved high-frequency wave modes from dominating the simulation. Currently, it remains an open question whether high-order numerical methods for nonlinear conservation laws can be simultaneously entropy-stable and locally energy-stable. In this presentation, we examine the local linear-stability properties of recently developed entropy-stable, high-order accurate dual-pairing (DP) SBP methods, as introduced by Stewart et al. [arXiv: 2411.06629], for nonlinear conservation laws. Our analysis indicates that the entropy-stable volume upwind filter inherent in these methods can ensure local linear stability. This approach offers a novel numerical strategy for designing reliable high-order methods for nonlinear conservation laws that are provably entropy-stable and locally energy-stable. The theoretical findings are supported by numerical experiments involving the inviscid Burgers' equation and nonlinear shallow water equations, in one and two space (2D) dimensions. Furthermore, we present accurate numerical simulations of 2D barotropic shear instability, with fully developed turbulence, demonstrating the efficiency of the DP SBP method in resolving turbulent scales.