Worst-Case Analysis, Safety Margins, and Fuzzy Algebra: A Mathematical Equivalence

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Classical structural engineering, fuzzy set theory, interval-based reasoning, and multivalued logic are often studied as separate subjects, each with its own language and methods. This work shows that, in the context of structural safety analysis, these areas are mathematically linked in a direct and rigorous way. The main objective is to demonstrate that the standard engineering notion of a safety margin can be interpreted exactly as a fuzzy membership function, and that the resulting framework naturally leads to both fuzzy intersection rules and multivalued truth values.

The key idea is to examine the safety margin of each structural member as a normalized measure of reserve capacity. For a given load configuration, the safety margin takes values between 0 and 1, where 0 corresponds to violation of the allowable stress limit and 1 corresponds to maximal safety within the chosen normalization. Under this interpretation, each member defines a fuzzy set of admissible loads. The associated α-cuts reproduce the classical allowable-load regions corresponding to prescribed safety factors. Thus, the usual engineering design requirement that a member must satisfy a given safety factor is equivalent to requiring that the load point belong to a particular α-cut of the corresponding fuzzy safety set.

For the structure as a whole, the global safety margin is obtained by taking the minimum of the memberwise safety margins. This coincides exactly with the standard fuzzy intersection operation. In this way, the worst-case criterion used in structural mechanics matches the basic aggregation rule of fuzzy algebra. The analysis therefore provides a mathematically transparent interpretation of structural safety as a fuzzy-set construction rather than merely a heuristic analogy.

The report also shows that the propagation of external loads through the structural equilibrium equations is equivalent to Zadeh's extension principle. Internal forces are determined as functions of the applied loads, and the safety margins induced by these forces arise from the same mathematical mechanism used in fuzzy mapping of uncertain or graded inputs. This interpretation also clarifies the dependency problem that appears when the same underlying variables are expressed in different but mathematically related forms, connecting the discussion to interval analysis and exact versus overestimated bounds.

A further contribution of this work is the interpretation of the global safety margin as a multivalued degree of truth. Instead of classifying a structure as simply safe or unsafe in a binary sense, the framework assigns a graded truth value to the statement that the structure is safe under the given load configuration. This yields a fully interpretable logical meaning for safety margins and provides a bridge between engineering design, fuzzy membership, and many-valued logic.

These ideas are illustrated in detail through the example of a five-bar planar truss subjected to two independent loads. The example makes the theory concrete by showing how safety regions, α-cuts, worst-case behavior, and graded truth values can all be visualized and computed within a single unified model. The overall result is a conceptually simple but mathematically strong framework that connects engineering safety analysis with fuzzy algebra and multivalued logic in a way that is both rigorous and physically meaningful.