Laplacian Eigenvalues of Bipartite Kneser-Like Graphs

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Bipartite Kneser-like graphs $G(a,b)$ are defined by $a, b \in \mathbb{N}$ such that $a > b$ and letting $n = a + b + 1$. Then, let $\mathcal{A}$ and $\mathcal{B}$ be a-sized and b-sized subsets of $[n]$, respectively. An edge is drawn between all vertices of $A \in \mathcal{A}$ and $B \in \mathcal{B}$ if and only if $A \cap B = \emptyset$. The Laplacian matrix of $G(a,b)$ is defined as $L(G(a,b)) = Deg(G(a,b)) - Adj(G(a,b))$ where $Deg(G(a,b))$ is the diagonal matrix of vertex degrees in $G(a,b)$ and $Adj(G(a,b))$ is the adjacency matrix of the graph. A general construction of these graphs' eigenvectors has been proven by a previous student, and now we aim to prove that we can always construct a basis of vectors using this combinatorial description of the eigenvectors.

We conjecture that a certain set of these eigenvectors, chosen using ballot permutations, form a basis for $L(G(a,b))$ for any a and b. The ingredients of the conjectured proof include a lexicographic ordering of the eigenvectors and the Gram matrix of inner products of the basis eigenvectors. Proving this would demonstrate that all Laplacian eigenvalues of these graphs are always non-negative integers.