Partitioning Posets into ω-Chains

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Let $(P, \leq)$ be a poset with no maximal element. One can show that there exists a family $\mathcal{S}$ of disjoint chains in P, each order-isomorphic to ω, such that for every $p \in P$, there is $C \in \mathcal{S}$ and $q \in C$ with $p \leq q$. In essence, $\mathcal{S}$ acts like a sort of spanning set for P. It is natural to ask when $\mathcal{S}$ can be taken to be a full partition of P so that P can be partitioned into ω-chains. In this talk I will discuss three results related to this question. The first shows that in the case when P is countably infinite, $\mathcal{S}$ can always be taken to be a full partition of P. The other two results concern forbidden subsets of P should there exist a partition of P into ω-chains. For $A \subseteq P$, we denote the strict upset of A in P by $A \uparrow = \{p \in P \mid p > q \text{ for some } q \in A\}$. If P can be partitioned into ω-chains, then for every subset Q of P, it must be that $|Q \uparrow| \geq |Q|$. Further, if there exists a subset K of P whose dual is well-founded, then it must be that $|K \uparrow - K| \geq |K|$.