Kernel Density Estimation and Convolution

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We revisit kernel density estimation (KDE) through a convolutional framework that provides a simple model-based interpretation and extends naturally to constrained domains. Motivated by this perspective, we propose **SHIDE** (Simulation and Histogram Interpolation for Density Estimation), a computationally efficient estimator that generates pseudo-data using bounded noise and applies spline interpolation to histograms.

The noise is drawn from bounded polynomial kernels formed by convolving uniform distributions, with bandwidth determined by the kernel support. We establish pointwise consistency, a bias–variance decomposition, and asymptotic mean integrated squared error (MISE), showing that SHIDE attains the classical $n^{-4/5}$ rate while reducing boundary bias.

We further develop two data-driven bandwidth selectors, an AMISE-optimal rule and a percentile-based rule, and show their asymptotic equivalence. Simulations demonstrate that SHIDE is competitive with KDE and often advantageous for bounded or heavy-tailed distributions.

Keywords: Bandwidth selection, Convolution, Histogram, Kernel density estimation (KDE), Pseudo-data, Spline interpolation.